

## Nonparabolicity of the conduction band of wurtzite GaN

S. Syed, J. B. Heroux, Y. J. Wang, M. J. Manfra, R. J. Molnar et al.

Citation: *Appl. Phys. Lett.* **83**, 4553 (2003); doi: 10.1063/1.1630369

View online: <http://dx.doi.org/10.1063/1.1630369>

View Table of Contents: <http://apl.aip.org/resource/1/APPLAB/v83/i22>

Published by the [American Institute of Physics](#).

---

### Related Articles

Interface trap generation and recovery mechanisms during and after positive bias stress in metal-oxide-semiconductor structures

*Appl. Phys. Lett.* **100**, 203503 (2012)

Analysis of vacancy-induced amorphization of single-layer graphene

*Appl. Phys. Lett.* **100**, 203105 (2012)

Scaling relations and the role of bond-charge to the electron transmission through two coupled Aharonov-Bohm rings

*J. Appl. Phys.* **111**, 094304 (2012)

Contactless electroreflectance studies of Fermi level position on c-plane GaN surface grown by molecular beam epitaxy and metalorganic vapor phase epitaxy

*Appl. Phys. Lett.* **100**, 181603 (2012)

Substrate effect on the electronic structures of CuPc/graphene interfaces

*Appl. Phys. Lett.* **100**, 161603 (2012)

---

### Additional information on *Appl. Phys. Lett.*

Journal Homepage: <http://apl.aip.org/>

Journal Information: [http://apl.aip.org/about/about\\_the\\_journal](http://apl.aip.org/about/about_the_journal)

Top downloads: [http://apl.aip.org/features/most\\_downloaded](http://apl.aip.org/features/most_downloaded)

Information for Authors: <http://apl.aip.org/authors>

## ADVERTISEMENT



**Goodfellow**  
metals • ceramics • polymers • composites  
70,000 products  
450 different materials  
**small quantities fast**

[www.goodfellowusa.com](http://www.goodfellowusa.com)

## Nonparabolicity of the conduction band of wurtzite GaN

S. Syed<sup>a)</sup>

*Department of Applied Physics and Applied Mathematics, Columbia University, New York, New York 10027*

J. B. Heroux

*Department of Electrical Engineering, Columbia University, New York, New York, 10027*

Y. J. Wang

*National High Magnetic Field Laboratory, Florida State University, Tallahassee, Florida 32306*

M. J. Manfra

*Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974*

R. J. Molnar

*MIT Lincoln Laboratory, Lexington, Massachusetts 02420-0122*

H. L. Stormer

*Department of Physics and Department Applied Physics and Applied Mathematics, Columbia University, New York, New York 10027*

(Received 18 July 2003; accepted 3 October 2003)

Using cyclotron resonance, we measure the effective mass,  $m^*$ , of electrons in AlGaIn/GaN heterostructures with densities,  $n_{2D} \sim 1$  to  $6 \times 10^{12} \text{ cm}^{-2}$ . From our extensive data, we extrapolate a band edge mass of  $(0.208 \pm 0.002)m_e$ . By comparing our  $m^*$  data with the results of a multiband  $\mathbf{k} \cdot \mathbf{p}$  calculation, we infer that the effect of remote bands is essential in explaining the observed conduction-band nonparabolicity (NP). Our calculation of polaron mass corrections—including finite width and two-dimensional (2D) screening—suggests those to be negligible. It implies that the behavior of  $m^*(n_{2D})$  can be understood solely in terms of NP. Finally, using our NP and polaron corrections, we are able to reduce the large scatter in the published band edge mass values. © 2003 American Institute of Physics. [DOI: 10.1063/1.1630369]

The magnitude of the nonparabolicity (NP) of the conduction band of wurtzite GaN currently remains controversial. NP of a band can be probed by measuring the effective mass  $m^*$  as a function of energy. Such experiments have been performed in the past in both bulk and in two-dimensional electron systems (2DES). The deduced band edge mass values,  $m_0^*$ , however, exhibit considerable scatter. Using cyclotron resonance (CR), Drechsler *et al.*<sup>1</sup> determined  $m_0^*$  in bulk wurtzite GaN to be  $0.20m_e$ , where  $m_e$  is the free electron mass. Other methods, such as infrared reflectivity on electron plasma<sup>2</sup> and spectroscopy on shallow donors<sup>3–5</sup> in bulk GaN have yielded  $0.220m_e < m_0^* < 0.236m_e$ . An even wider range of values for the band edge mass,  $0.185m_e < m_0^* < 0.231m_e$ , emerge from experiments in AlGaIn/GaN heterostructures. From the temperature dependence of Shubnikov-de Haas (SdH) oscillations of 2DES, Lin *et al.*<sup>6</sup> deduced  $m_0^* = 0.22m_e$  while Hang *et al.*<sup>7</sup> reported  $m_0^* = 0.185m_e$ . CR experiments on heterostructures have revealed  $0.223m_e < m_0^* < 0.231m_e$ .<sup>8,9</sup> This spread in  $m_0^*$  suggests that the “extrapolation” from the various experiment values  $m^*(E)$  to the band edge remains poorly controlled.

Using CR, we have measured  $m^*$  in a series of high mobility ( $\mu \sim 20\,000 \text{ cm}^2/\text{V s}$ ) AlGaIn/GaN structures. Our heterostructures are all grown by molecular-beam epitaxy (MBE) on GaN templates prepared by hydride vapor phase epitaxy (HVPE). The specimens are described in detail elsewhere.<sup>10,11</sup> Our data cover a density range from 1 to 6

$\times 10^{12} \text{ cm}^{-2}$ . In these two-dimensional (2D) systems, this implies energies from  $\sim 27 \text{ meV}$  to  $\sim 120 \text{ meV}$  above the band edge due to electron confinement and band filling. Therefore, our mass data probe the NP of the GaN conduction band in small steps over a wide energy range. We also perform extensive  $\mathbf{k} \cdot \mathbf{p}$  calculations and determine that instead of the commonly used two-band model a multiband model is required to explain our experimental results. Additionally, our calculations on polaron correction of the effective mass of 2D electrons in GaN show them to be at the 1% level, considerably less than previously thought.<sup>8</sup> A detailed comparison between our data and  $\mathbf{k} \cdot \mathbf{p}$  calculation sets the band mass value to  $m_0^* = (0.208 \pm 0.002)m_e$  and, when applied to the results of other investigators, considerably reduces the spread in band edge mass values.

A Fourier transform spectrometer with light pipe optics and a composite Si bolometer was used for the detection of far-infrared transmission. A magnetic field was applied normal to the 2D electron layer. The carrier density of each sample was determined *in situ* from the SdH oscillations of the 2DES. All CR and SdH experiments were conducted at 4.2 K.

Figure 1 shows the CR energies versus magnetic field,  $B$  of a sample with  $n_{2D} = 2.3 \times 10^{12} \text{ cm}^{-2}$ . All data are taken with a resolution of 0.24 meV. The inset of Fig. 1 shows high-field spectra normalized to the spectrum taken at  $B = 0 \text{ T}$ . The solid line in Fig. 1 is a fit to the data at high fields,  $B > 27 \text{ T}$ , and low fields,  $B < 12 \text{ T}$ , resulting in an effective mass of  $m^* = 0.228m_e$ . For  $15 \text{ T} < B < 25 \text{ T}$ , there is

<sup>a)</sup>Electronic mail: sheyum@phys.columbia.edu

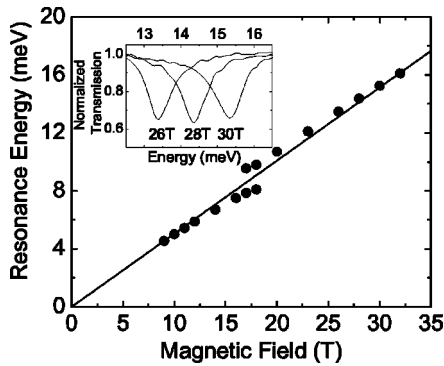


FIG. 1. Resonance energies vs magnetic field,  $B$ , of a sample with  $n_{2D} = 2.3 \times 10^{12} \text{ cm}^{-2}$  taken with a resolution of 0.24 meV. High- ( $B > 27 \text{ T}$ ) and low- ( $B < 12 \text{ T}$ ) field resonances can be fit with a single straight line, giving an effective mass of  $0.228m_e$ . Near 18 T, a level anticrossing results in a splitting of the CR. Inset: Transmission data for  $B = 26, 28,$  and  $30 \text{ T}$ , normalized to the spectrum at  $B = 0 \text{ T}$ .

a pronounced deviation from this straight line. This anomaly in the CR is being analyzed and published elsewhere.<sup>10</sup> Here, we observe that this anomaly is limited to a finite field region outside which all CR data can be fit by a straight line.

We have measured the effective mass in 11 samples with carrier density,  $n_{2D}$ , ranging from 1 to  $6 \times 10^{12} \text{ cm}^{-2}$ . In all cases, we observed either a broadening or a splitting of the CR line at intermediate fields but could fit the data away from this regime as well as the data seen in Fig. 1. Figure 2 shows the dependence of  $m^*$  on  $n_{2D}$ . For comparison, we also plot data from Refs. 8, 9, and 12. The mass data from the different references are located in the general vicinity of our results but, due to their considerable error bar or sparsity, are difficult to extrapolate to zero density, i.e., to the conduction-band minimum. The combined data of Fig. 2 show an increase in  $m^*$  by  $\sim 17\%$  as  $n_{2D}$  changes from 1 to  $9 \times 10^{12} \text{ cm}^{-2}$ . The rise in  $m^*$  with  $n_{2D}$  reflects the NP of the conduction band of the GaN host. Simply extrapolating our closely spaced data linearly to vanishing  $n_{2D}$ , we arrive at a band edge mass of  $m_0^* = 0.214m_e$ . This value is about

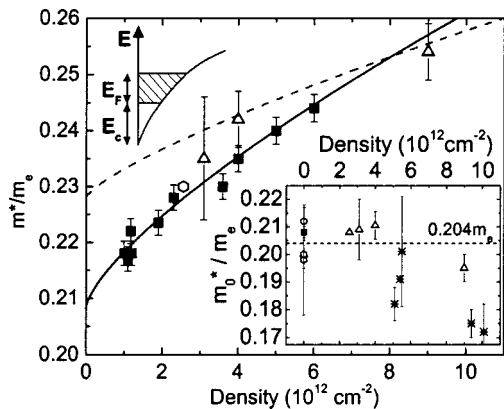


FIG. 2. Effective mass,  $m^*$ , in AlGaIn/GaN systems vs 2D electron density. Our data are shown as solid squares. Data from Refs. 8, 9, and 12 are shown as open symbols. The dashed line is a two band fit to the results of Ref. 9 according to Eq. (1) with  $K=1$ . The solid line represents a fit with  $K=2.5$ , which accounts for the influence of additional, higher conduction bands. Inset: Values of the band edge, mass from Refs. 1–9, 12, and this work, after NP and polaron corrections. CR results are shown as open triangles, donor spectroscopy data as open hexagons and SdH data as stars. Our result of  $m_0^* = (0.208 \pm 0.002)m_e$  is shown as a solid square. Averaging all data (except SdH) yields  $m_0^* = 0.204m_e$  illustrated by a dashed line.

$\sim 8\%$  lower than previously published CR data.<sup>8,9</sup> Since our data contain a small error bar and extend to very low  $n_{2D}$ , such a simple extrapolation should already be quite reliable.

Other groups have previously addressed NP in GaN. For example, Knap *et al.*<sup>8,9</sup> explored NP using CR in AlGaIn/GaN systems with different  $n_{2D}$  (see Fig. 2) and accounted for the magnitude of NP using a simple two-band approximation. In such an approximation, which includes only coupling between the lowest conduction and highest valence bands, the effective mass varies as

$$m^*(E) = m_0^*(1 + 2KE/E_g), \quad (1)$$

with  $K=1$  and  $E_g=3.5 \text{ eV}$ . In a 2DES, the energy,  $E = E_k + E_F$ , above the band minimum is composed of the average kinetic energy,  $E_k$  of the electrons in the confining potential well and  $E_F = \hbar^2 \pi n_{2D} / m_0^*$ ,<sup>13</sup> the Fermi energy of the 2DES. The value of  $E_k$  is dependent on the form of the wave function of the confined electrons. In a simple triangular potential approximation, the average kinetic energy is  $E_k = E_c/3$ , with  $E_c$  being the confinement energy of the lowest subband measured from the bottom of the conduction band.<sup>14</sup> Using the more accurate Fang–Howard variational wave function gives  $E_k = \hbar^2 b^2 / 8 m_0^*$ , where  $b^3 = 48 \pi m_0^* e^2 (N_{\text{dep}} + 11/32 n_{2D}) / \epsilon \hbar^2$ .<sup>14</sup> In our samples, since both the MBE and the HVPE GaN are  $n$  type, the depletion layer density,  $N_{\text{dep}}$ , can be set to zero. We use this two-band model with only one adjustable parameter,  $m_0^*$ , to fit the high-density data of Ref. 9 (dashed line in Fig. 2).  $E_k$  was computed in the triangular approximation following the authors of Ref. 9. Clearly, although this procedure can describe the original density dependence of  $m^*$  of Ref. 9 due to the rather large error bars, it fails to account for our own data. Neither a simple vertical shift of the line (a different  $m_0^*$ ) nor the usage of the Fang–Howard wave function can resolve this discrepancy. What is required, is a much stronger dependence of  $m^*$  on  $n_{2D}$ .

Empirically, we pursue an approach taken by Singleton *et al.*,<sup>15</sup> who used a *modified* two-band model to describe their NP data in GaAs. The authors considered  $K$  in Eq. (1) to be a second fitting parameter. The inclusion of a variable  $K > 1$  into the analytic expression incorporates the influence of higher conduction bands, simulating the results of a more elaborate, multiband  $\mathbf{k} \cdot \mathbf{p}$  calculation.<sup>16</sup> Working with the Fang–Howard model to determine  $E_k$ , we find a very good fit to our data for  $K=2.5$  and  $m_0^* = (0.208 \pm 0.002)m_e$ . Even if we use the less reliable triangular approximation, the required  $K=1.9$ . The effectiveness of the modified expression in fitting the density dependence of  $m^*$  over a wide range of  $n_{2D}$  demonstrates that the conduction band of GaN is more nonparabolic than was previously assumed.<sup>6–9</sup>

Before concluding that the proposed NP model appropriately describes our CR data, we need to assure ourselves that polaron effects are a negligible contributor to  $m^*$ . This mass enhancement factor results from electron–longitudinal optical (LO) phonon coupling in polar semiconductors, such as GaN. A determination of a polaron mass enhancement in heterostructures requires inclusion of screening in 2D and the finite width of the electronic wave function,<sup>17</sup> since both greatly reduce interaction between 2D carriers and LO phonons.<sup>18</sup> Following Ref. 18, we calculated the polaron effective mass in AlGaIn/GaN heterostructures using a Fang–



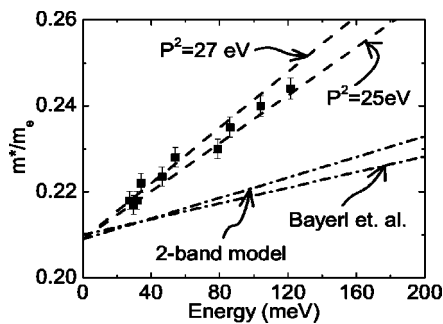


FIG. 3. Variation of  $m^*$  with energy according to  $\mathbf{k}\cdot\mathbf{p}$  calculations, assuming a band edge mass of  $0.208m_e$ . The two-band model and the five-band  $\mathbf{k}\cdot\mathbf{p}$  results of Bayerl *et al.* (see Ref. 20), which neglect remote bands, both underestimate the energy dependence of  $m^*$ . The experimental data (solid squares) can be explained by multiband  $\mathbf{k}\cdot\mathbf{p}$  calculations, that include the influence of remote bands.

Howard variational wave function, a static Thomas–Fermi screening model, and a Frohlich constant,  $\alpha=0.49$ .<sup>1</sup> We find a polaron enhancement of less than 1% for  $m^*$  for  $n_{2D}=9\times 10^{12}\text{ cm}^{-2}$ . Since the effect decreases with decreasing density, corrections for lower-density specimens are smaller yet. This mass enhancement is considerably smaller than the 10% estimated previously for  $n_{2D}=3.1\times 10^{12}\text{ cm}^{-2}$ , where screening and finite width had been neglected.<sup>8</sup> A 1% mass enhancement, due to polaronic coupling, lies within the error bars of our CR data.

The large spread in the value of the band edge mass  $m_0^*$  in literature is mostly due to two reasons: The underestimation of the NP and the overestimation of polaronic corrections. Applying our NP and polaron corrections to the available effective mass data (Refs. 1–9 and 12), we reach a much more coherent picture for the band edge effective mass,  $m_0^*$ , in GaN, as shown in the inset to Fig. 2. We observe that the majority of the values for  $m_0^*$  are very close to an average  $m_0^*=0.204m_e$ . However, most of the  $m^*$  data from SdH (displayed in stars in Fig. 2) remain at variance from the CR, infrared reflectivity, and donor spectroscopy data, a fact that remains unexplained.

Since  $m^*(E)$  cannot be accurately represented with a two-band model, a five-band  $\mathbf{k}\cdot\mathbf{p}$  calculation<sup>16</sup> in the zincblende approximation was performed to model our data. Figure 3 shows the results. The material parameters employed, using the Koster notation, are the spin–orbit splitting,  $\Delta'_0$ , of the  $\Gamma_5$  conduction band and the momentum matrix elements,  $P^2$  and  $\lambda^2P^2$ , coupling the  $\Gamma_1$  conduction band with the  $\Gamma_5$  valence and conduction bands. They are not precisely known. The formalism proposed by Carlos *et al.*<sup>19</sup> and used by Bayerl *et al.*<sup>20</sup> in a five-band model to relate the parameters was utilized to calculate  $m^*(E)$ . However, as seen in Fig. 3, the parameters chosen by Bayerl *et al.* lead to an NP even lower than in a two-level model. We find that the experimental data points can only be matched if an extra parameter  $C$ , taking remote bands at  $k=0$  into account, is included.<sup>21</sup> If one assumes  $C=-1.5$  (compare with  $C=-2$  for GaAs),<sup>16</sup>  $P^2$  is in the range 25–27 eV with  $\lambda^2$

$\approx 0.33-0.48$  and  $\Delta'_0\approx 120-180\text{ meV}$ . This set of parameters is close to a good set of values chosen by Kennedy<sup>22</sup> and confirm that  $P^2$  is relatively large for GaN ( $P^2\approx 26\text{ eV}$ ).

In conclusion, our CR experiments set the conduction band edge mass in GaN to  $m_0^*=0.208\pm 0.002m_e$ . Using our determination of the NP and reviewing the polaron correction, we reach much better agreement between several published data for  $m_0^*$ .

The authors thank M. S. Brandt, L. N. Pfeiffer, K. W. West, and W. I. Wang for helpful discussions. A portion of the work was performed at the National High Magnetic Field Laboratory, which is supported by NSF Cooperative Agreement No. DMR-0084173 and by the State of Florida. Financial support from the W. M. Keck Foundation is gratefully acknowledged.

- <sup>1</sup>M. Drechsler, D. M. Hofmann, B. K. Meyer, T. Detchprohm, H. Amano, and I. Asasaki, *Jpn. J. Appl. Phys., Part 1* **34**, L1178 (1995).
- <sup>2</sup>P. Perlin, E. Litwin-Staszewska, B. Suchanek, W. Knap, J. Camassel, T. Suski, R. Piotrkowski, I. Grzegory, S. Porowski, E. Kaminska, and J. C. Chervin, *Appl. Phys. Lett.* **68**, 1114 (1996).
- <sup>3</sup>W. J. Moore, J. A. Freitas, Jr., and R. J. Molnar, *Phys. Rev. B* **56**, 12073 (1997).
- <sup>4</sup>A. M. Witowski, K. Pakula, J. M. Baranowski, M. L. Sadowski, and P. Wyder, *Appl. Phys. Lett.* **75**, 4154 (1999).
- <sup>5</sup>B. K. Meyer, D. Vohm, A. Graber, H. C. Alt, T. Detchprohm, A. Amano, and I. Asasaki, *Solid State Commun.* **95**, 597 (1995).
- <sup>6</sup>T. Y. Lin, H. M. Chen, M. S. Tsai, Y. F. Chen, F. F. Fang, C. F. Lin, and G. C. Chi, *Phys. Rev. B* **58**, 13793 (1998).
- <sup>7</sup>D. R. Hang, C.-T. Liang, C. F. Huang, Y. H. Chang, Y. F. Chen, H. X. Jiang, and J. Y. Lin, *Appl. Phys. Lett.* **79**, 66 (2001).
- <sup>8</sup>W. Knap, S. Contreas, H. Alause, C. Skierbiszewski, J. Camassel, M. Dyakonov, J. Yang, M. Asif Khan, Q. Chen, J. Yang, M. L. Sadowski, S. Huan, F. H. Yang, M. Goiran, J. Leotin, and M. S. Shur, *Appl. Phys. Lett.* **70**, 2123 (1997).
- <sup>9</sup>W. Knap, H. Alause, J. M. Bluet, J. Camassel, J. Young, M. Asif Khan, Q. Chen, S. Huan, and M. Shur, *Solid State Commun.* **99**, 195 (1996).
- <sup>10</sup>S. Syed, M. J. Manfra, Y. J. Wang, H. L. Stormer, and R. J. Molnar, *Phys. Rev. B* **67**, 241304 (2003).
- <sup>11</sup>M. J. Manfra, N. G. Weimann, J. W. P. Hsu, L. N. Pfeiffer, K. W. West, S. Syed, H. L. Stormer, W. Pan, D. V. Lang, S. N. G. Chu, G. Kowach, A. M. Sergent, J. Caissie, K. M. Molnar, L. J. Mahoney, and R. J. Molnar, *J. Appl. Phys.* **92**, 338 (2002).
- <sup>12</sup>Y. J. Wang, R. Kaplan, H. K. Ng, K. Doverspike, D. K. Gaskill, T. Ikedo, I. Asasaki, and H. Amano, *J. Appl. Phys.* **79**, 8007 (1996).
- <sup>13</sup>Here we use an average mass of  $m_0^*=0.23m_e$ , since it enters  $E_F$  as  $(m_0^*)^{-1}$  and  $E_k$  as  $(m_0^*)^{-1/3}$  and therefore enters  $m^*$  in higher order.
- <sup>14</sup>T. Ando, F. B. Fowler, and F. Stern, *Rev. Mod. Phys.* **54**, 437 (1982).
- <sup>15</sup>J. Singleton, R. J. Nicholas, D. C. Rogers, and C. T. B. Foxon, *Surf. Sci.* **196**, 429 (1988).
- <sup>16</sup>P. Pfeiffer and W. Zawadzki, *Phys. Rev. B* **41**, 1561 (1990).
- <sup>17</sup>According to S. Das Sarma, *Phys. Rev. B* **27**, 2590 (1983); phase-space restrictions due to the degeneracy of the 2DES can be ignored for  $k_F/q_0$  at least as high as 0.6, where  $q_0$  is the phonon wave vector. In our AlGaIn/GaN systems  $k_F/q_0\leq 0.6$  for  $n_{2D}\leq 3.4\times 10^{12}\text{ cm}^{-2}$  and still  $k_F/q_0\approx 0.8$  for our highest density. We, therefore, also ignore phase-space restriction.
- <sup>18</sup>S. Das Sarma, *Phys. Rev. B* **27**, 2590 (1983).
- <sup>19</sup>W. E. Carlos, J. A. Freitas Jr., M. A. Khan, D. T. Olson, and J. N. Kuznia, *Phys. Rev. B* **48**, 17878 (1993).
- <sup>20</sup>M. W. Bayerl, M. S. Brandt, T. Graf, O. Ambacher, J. A. Majewski, M. Suitsmann, D. J. As, and K. Lischka, *Phys. Rev. B* **63**, 165204 (2001).
- <sup>21</sup>C. Hermann and C. Weisbuch, *Phys. Rev. B* **15**, 823 (1977).
- <sup>22</sup>T. A. Kennedy, E. R. Glaser, W. E. Carlos, P. P. Ruden, and S. Nakamura, *MRS Internet J. Nitride Semicond. Res.* **4S1**, G7.4 (1999).