

## Gapped excitations of unconventional fractional quantum Hall effect states in the second Landau level

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We report the observation of low-lying collective charge and spin excitations in the second Landau level at  $\nu = 2 + 1/3$  and also for the very fragile states at  $\nu = 2 + 2/5$  and  $2 + 3/8$  in inelastic light scattering experiments. These modes exhibit a clear dependence on filling factor and temperature substantiating the unique access to the characteristic neutral excitation spectra of the incompressible fractional quantum Hall effect (FQHE) states. A detailed mode analysis reveals low-energy modes at around  $70 \mu\text{eV}$  and a sharp mode slightly below the Zeeman energy interpreted as gap and spin-wave excitation, respectively. The lowest-energy collective charge excitation spectrum at  $\nu = 2 + 1/3$  exhibits significant qualitative similarities with its cousin state in the lowest Landau level at  $\nu = 1/3$  suggesting similar magnetoroton minima in the neutral excitations. The mode energies differ by a scaling of 0.15 indicating different interaction physics in the  $N = 0$  and  $N = 1$  Landau levels. The striking polarization dependence in elastic and inelastic light scattering is discussed in the framework of anisotropic electron phases that allow for the stabilization of nematic FQHE states. The observed excitation spectra provide new insights by accessing quantum phases in the bulk of electron systems and facilitate comparison with different theoretical descriptions of those enigmatic FQHE states.

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Ultraclean two-dimensional electron systems subjected to high perpendicular magnetic fields form diverse quantum ground states that are driven by strong Coulomb interactions between electrons. In a partially populated  $N = 0$  Landau level (LL) fractional quantum Hall effect (FQHE) states are interpreted as weakly interacting quasiparticles of electrons with even numbers of vortices of the many-body wave function attached to the electrons [known as composite fermions (CFs)] [1]. The quantum phases in higher LLs ( $N > 1$ ) are governed by different interaction physics [2–4]. The second Landau Level (SLL) with  $N = 1$  is special since odd-denominator FQHE states as well as unconventional FQHE states such as the enigmatic even-denominator states at  $\nu = 5/2$  [5] and  $\nu = 7/2$  compete with other ground states. Competing phases manifest in transport experiments in an anisotropic longitudinal resistance and as reentrant integer quantum Hall effect [6–8]. For  $\nu = 2 + 1/3 = 7/3$  a large anisotropy in the resistance and a robust FQHE state are in coexistence indicating that the FQHE can be stabilized in absence of full rotational invariance [9–12]. It has been proposed that transport anisotropies in the SLL can be explained in terms of nematic electron liquid, a compressible metallic phase that is expected to exhibit strong signatures in polarized light scattering experiments due to unequal longitudinal and transverse susceptibilities  $\chi_L$  and  $\chi_T$  [13].

The nature of both the more conventional as well as unconventional FQHE states in the SLL are not yet well known. Similarly, their low-lying collective excitation spectra, unique fingerprints of each state, are neither theoretically well understood nor experimentally observed. The excitations of the FQHE state at  $\nu = 2 + 1/3$ , the cousin of the most robust state at  $\nu = 1/3$ , are predicted as composite fermions dressed with an exciton cloud [14]. Balram *et al.* [14] state that the  $1/3$  and  $2+1/3$  could be determined by the same physics and the exciton screening impacts the  $2 + 1/3$  state only quantitatively without changing its nature. Besides the much studied FQHE state at  $\nu = 5/2$ , interpreted as a  $p$ -wave paired state of composite fermions supporting non-Abelian excitations [15], the state at  $2 + 2/5$  is envisioned as an exotic parafermionic state [16]. It has been suggested that the weaker  $2 + 2/5$  and  $2 + 3/8$  FQHE states exhibit even greater potential than the  $5/2$  state to serve as a model system for fault-tolerant quantum computation [16,17].

Collective charge and spin excitations of phases in the  $N = 0$  lowest LL (LLL) are accessed by resonant inelastic light scattering (RILS) methods [18–22], and quantitative comparisons of the measured low-lying excitation spectra with theory provide in-depth understanding of the physics driving the emergence of those quantum states [23,24]. Interpretations of measured low-lying excitations in the  $N = 1$  LL from theoretical formulations of the underlying quantum phases offer further insights into interaction physics in the SLL.

In this Rapid Communication we report RILS observations of a remarkable filling factor dependence of low-lying

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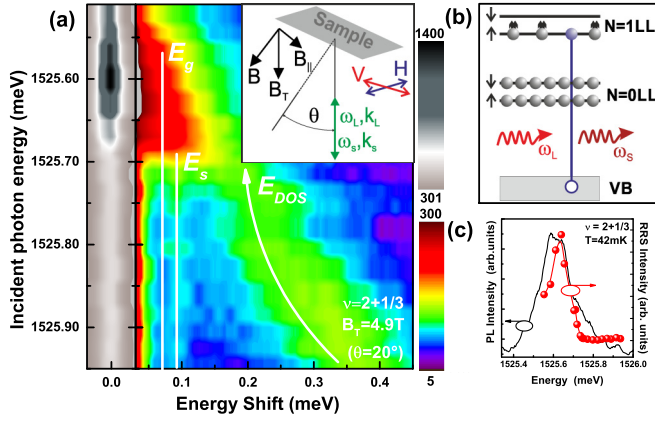


FIG. 1. (Color online) (a) Color plot of RRS (gray scale) and RILS intensities (color scale) for photon energies close to the optical emission from the  $N = 1$  LL at  $\nu = 2 + 1/3$  as a function of the exciting light energy  $\omega_L$  measured at temperature  $T = 42$  mK. The marked dependence of line shapes on  $\omega_L$  is due to a strong outgoing resonance in RILS [25]. Three modes are seen at energies  $E_s$ ,  $E_{DOS}$ , and  $E_g$ . Inset: Light scattering geometry and the magnetic field direction. The red and blue arrows denote the linear polarization of photons. (b) Energy level scheme for incoming and outgoing photon energies ( $\omega_{L(S)}$ ) resulting in resonant enhancement in RRS and RILS spectra. The energy is close to the optical transition between valence band and spin-up branch of the  $N = 1$  LL. (c) Emission spectrum from the  $N = 1$  LL (black line) and related RRS intensities (red dots) obtained from the spectra shown in (a).

excitations of the partially populated SLL in the range  $5/2 > \nu > 2 + 1/5$ . RILS spectra are interpreted in terms of density of states of large wave-vector modes that are activated by residual disorder. The modes exhibit a marked filling factor dependence and are only well developed for filling factors that are known from transport to form incompressible, albeit weak FQHE states such as  $\nu = 2 + 2/5$ ,  $2 + 3/8$ , and  $2 + 1/3$  [6,26,27]. Energy gaps identified in these measurements are well below 0.1 meV (about 1 K). These observations suggest that the FQHE states seen in transport in the filling factor range  $2 + 2/5 > \nu > 2 + 1/5$  have well-defined low-lying gapped excitation modes that manifest the underlying interaction physics in the SLL.

We find that RILS spectra at the filling factors of these weak FQHE states typically display three distinct modes with intensity that is resonantly enhanced as shown in Fig. 1(a) for  $\nu = 2 + 1/3$ , the most robust odd-denominator FQHE state in the SLL. In Fig. 1(a) there is a band with a maximum that shifts with  $\omega_L$  and occurs in the energy range from  $0.15 \text{ meV} < E_{DOS} < 0.35 \text{ meV}$ . In addition there is a broad mode centered at  $E_g \approx 0.08 \text{ meV}$ , and a weak sharp mode at  $E_s \approx 0.1 \text{ meV}$ . The modes  $E_{DOS}$  and  $E_g$  are interpreted as spin conserving excitations of the quantum fluid. The  $E_s$  mode is assigned to a low-lying excitation with spin reversal [28]. Comprehensive mode analysis at  $2 + 1/3$  uncovers that the mode labeled  $E_g$  can be decomposed into two modes as will be described below. A quantitative comparison with the calculated as well as measured excitation spectrum of the  $1/3$  state [23] indicates remarkable qualitative agreement between the lowest-energy mode dispersion for the  $2 + 1/3$

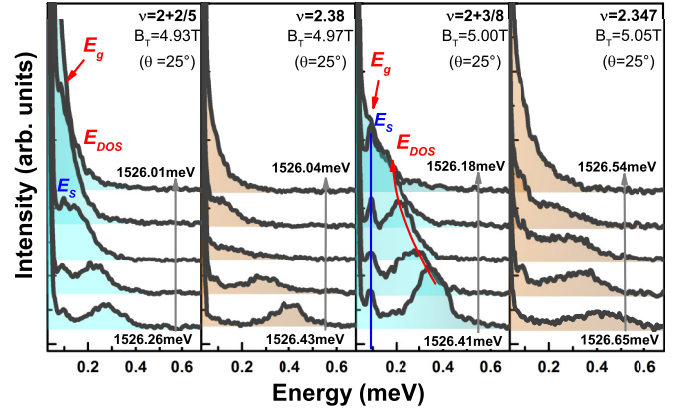


FIG. 2. (Color online) Filling factor dependence of RILS spectra for  $2 + 2/5 \geq \nu \geq 2.347$ . The spectra are shifted vertically for clarity. All observed, resonantly enhanced modes exhibit a striking filling factor dependence, drawing attention to the filling factors  $\nu = 2 + 2/5$  and  $\nu = 2 + 3/8$  that are known from transport to be incompressible FQHE states [(H, V),  $T = 42$  mK,  $\theta = 25^\circ$ ].

and  $1/3$  states. The mode energies in the SLL are reduced by a constant scaling of  $0.15 \pm 0.01$ . This finding suggests similar magnetoroton features in the neutral gap excitations at  $\nu = 2 + 1/3$  state and at its cousin state  $\nu = 1/3$  in the LLL. The energy might be lowered by an exciton cloud dressing CF quasiparticles effectively altering the CF interaction strength [14].

By tuning the filling factor away from the magic filling fractions the charge modes  $E_g$  and  $E_{DOS}$  almost disappear and the spin mode  $E_s$  is significantly reduced as shown in Fig. 2 and in the Supplemental Material [29]. Surprisingly, even the unconventional FQHE states at  $\nu = 2 + 2/5$  and  $2 + 3/8$ , known to be fragile in activated transport [6,26,27], exhibit well-defined low-energy modes in RILS spectra measured at the elevated temperature of  $T = 42$  mK. The distinct dependence on filling factor as well as on temperature of the three low-lying modes in Fig. 2 and the Supplemental Material [29,30] substantiate the link to incompressible quantum states. Interestingly, we observe a pronounced dependence of the RILS modes on photon polarization, which is most remarkable for the lowest-energy mode  $E_g$  (see Fig. 5). This observation is linked to the occurrence of nematic liquids induced by the application of finite in-plane magnetic fields [9,11–13]. The experimentally explored low-lying excitation spectra of the puzzling  $2 + 1/3$ ,  $2 + 2/5$ , and  $2 + 3/8$  FQHE states pave the way to distinguish between different scenarios about their nature provided by theory.

The ultraclean two-dimensional electron system is confined in a 30-nm-wide symmetrically doped single GaAs/AlGaAs quantum well structure. The charge carrier density and mobility determined from transport at  $T = 300$  mK are  $2.9 \times 10^{11} \text{ cm}^{-2}$  and  $23.9 \times 10^6 \text{ cm}^2/\text{Vs}$ , respectively. The measurements have been done in a  $^3\text{He}/^4\text{He}$  dilution refrigerator with a 16 T magnet and bottom windows for optical access. The RILS and resonant Rayleigh scattering (RRS) spectra are excited by a Ti:sapphire laser at a power below  $10^{-4} \text{ W/cm}^2$ . The energy of the light  $\omega_L$  is tuned to be close to the optical emission from the  $N = 1$  LL as sketched in

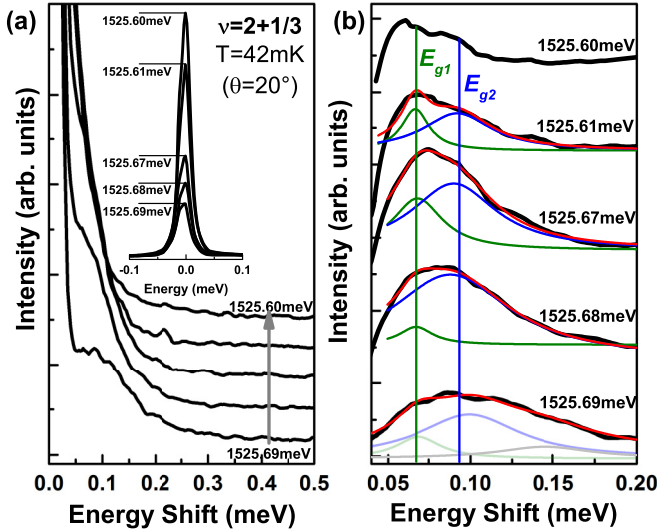


FIG. 3. (Color online) Mode analysis of the resonantly enhanced low-energy mode  $E_g$  for  $\nu = 2 + 1/3$  [ $(H, V)$ ,  $T = 42$  mK,  $\theta = 20^\circ$ ]. (a) Individual RILS spectra excited with photon energies  $\omega_L$ , vertically shifted for clarity. The inset displays the same spectra around zero energy highlighting the RRS contribution. (b) Spectra shown in (a) after subtraction of the RRS intensities by a Lorentz profile centered at  $\omega_L$ . A mode analysis with two Lorentzian uncovers two resonantly enhanced low-energy modes at  $E_{g1} \approx 67 \mu\text{eV} = 7.9 \times 10^{-3} E_c$  and  $E_{g2} \approx 90 \mu\text{eV} = 1.06 \times 10^{-2} E_c$ , respectively, with  $E_c = e^2/\epsilon l$  and  $l$  the magnetic length.

Fig. 1(b) to achieve resonant enhancement [28,31]. Emission from the  $N = 1$  LL and RRS spectra are displayed in Fig. 1(c). The used backscattering geometry is sketched in the inset of Fig. 1(a). The sample is tilted at an angle  $\theta = 20^\circ$  or  $\theta = 25^\circ$  in two different cooldowns, respectively, to allow the transfer of a finite momentum  $k = |\vec{k}_L - \vec{k}_S| = (2\omega_L/c) \sin \theta$ , where  $\vec{k}_{L(S)}$  is the in-plane component of the incident (scattered) photon,  $\omega_L$  the incoming photon energy, and  $c$  the speed of light. The tilt angle results in a small in-plane magnetic field component  $B_{\parallel}$  that still allows well-defined FQHE states at  $\nu = 5/2$  and  $\nu = 2 + 1/3$  and the formation of anisotropic phases in the second LL [8,9,32–34]. The spectra taken at different cooldowns with slightly different tilt angles of  $\theta = 20^\circ$  and  $\theta = 25^\circ$  are apparently looking very similar (compare, e.g., spectra displayed in Figs. 1, 3, and the Supplemental Material [29]). The filling factor as a function of magnetic field is precisely determined from the maximum of the spin-wave intensity, an excitation occurring at the bare Zeeman energy  $E_Z$ , for  $\nu = 3$  [28,35].

The polarization of incoming and scattered light is denoted with V and H as sketched in the inset of Fig. 1(a) and described in detail in the Supplemental Material [36]. Figure 1(a) the (HV) RILS spectra display three features that are interpreted as collective excitation modes of the incompressible quantum fluid at  $\nu = 2 + 1/3$ . Intensity maxima are assigned either to critical points in the wave-vector dispersion with a high density of state (DOS), such as rotons or maxons activated by breakdown of wave-vector conservation due to residual disorder [24], or to long wavelength modes with  $k = q$  [18]. In this framework the sharp mode  $E_s$  is interpreted as wave-vector

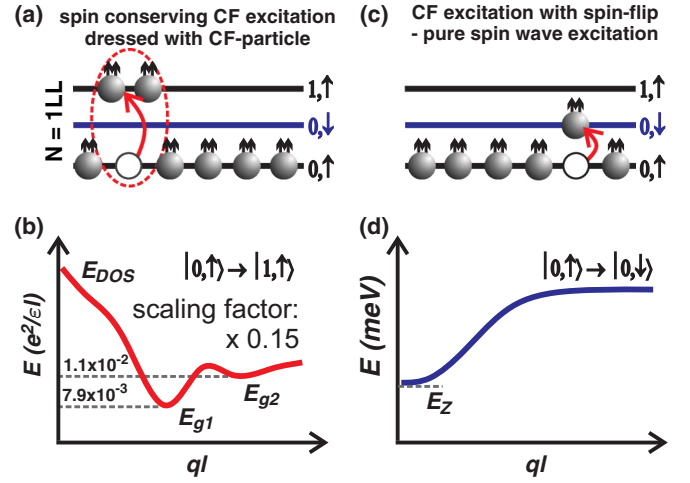


FIG. 4. (Color online) Spin-split  $\Lambda$  levels, which are Landau levels of CFs, within the  $N = 1$  LL and pictorial description of CF charge excitation consisting of a CF quasiparticle “dressed” with a spin-conserving CF excitation [14] in (a) and a spin-wave excitation (c). The related wave-vector dispersions are scaled down from the calculated dispersion of  $\nu = 1/3$  state in the LLL in panels (b) and (d), respectively (modified from [23] and [24]).

conserving  $q \rightarrow 0$  spin-wave excitation at the bare Zeeman energy  $E_Z = \mu_B g B$ , where  $\mu_B$  is the Bohr magneton and  $g$  the bare  $g$  factor. The deviation of the measured  $g$  factor  $|g| = 0.36$  from the value for free electrons in bulk GaAs ( $|g| = 0.44$ ) has previously been reported [28] and can be seen as a precursor for breakdown of full rotational invariance [37,38]. The mode  $E_s$  exhibits a less pronounced temperature dependence. The peak is slightly broadened at 250 mK and still observable at 600 mK consistent with the interpretation of  $E_s$  as a pure spin-wave mode that is broadened by increasing the temperature, but not much affected by melting of an incompressible fluid. The related excitation scheme and mode dispersion are depicted in Figs. 4(c) and 4(d).

The intense mode labeled  $E_g$  in RILS spectra at  $\nu = 2 + 1/3$  (Fig. 3) is regarded as superposition of the RILS signal and of a strong RRS [plotted in the inset of Fig. 3(a) and in Fig. 1(a)]. The two contributions to the light scattering intensities can be decomposed by subtracting the RRS signal that is well described by a single Lorentzian centered at  $\omega_L$ . The subtracted spectra shown in Fig. 3(b) reveal that at low energy the RILS component of the measured spectra can be well described by two Lorentzian peaks centered at  $E_{g1} \approx 67 \mu\text{eV} = 7.9 \times 10^{-3} E_c$  and  $E_{g2} \approx 90 \mu\text{eV} = 1.06 \times 10^{-2} E_c$ , respectively, with  $E_c = e^2/\epsilon l$  and  $l$  the magnetic length. The two contributions exhibit a slightly different resonant enhancement profile in RILS. The  $E_{g2}$  mode is resonantly enhanced for a smaller incoming photon energy  $\omega_L$  compared to the  $E_{g1}$  mode verifying that the two observed modes are indeed due to RILS by collective excitations.

We interpret the two modes  $E_{g1(2)}$  as lowest-energy collective spin-preserving neutral excitations of the  $\nu = 2 + 1/3$  FQHE state as sketched for CF quasiparticles in Fig. 4(a). Within the framework of breakdown of wave-vector conservation, the modes are expected to occur at critical points in



the wave-vector dispersion [23,24] and are assigned to a roton minimum  $\delta_R$  at finite  $q$  and to the large momentum limit  $\Delta_\infty$  at  $q \rightarrow \infty$  as depicted in Figs. 4(b) and 4(d). Similarly, the RILS intensity at  $E_{DOS}$  with a low-energy onset at around  $0.15 \text{ meV} \approx 1.76 \times 10^{-2} E_c$  is attributed to the mode  $\Delta_0$  at the long wavelength limit  $q \rightarrow 0$ . The quantitative interpretations in Figs. 4(b) and 4(d) are based on a scaling of the wave-vector dispersion of the well understood neutral gap excitation of the cousin state at  $\nu = 1/3$  in the LLL. It is striking that a very good quantitative interpretation of RILS spectra at  $\nu = 2 + 1/3$  is obtained by scaling down the mode of the  $\nu = 1/3$  dispersion by a constant factor of  $0.15 \pm 0.01$ . This result indicates that the  $q$  dispersion of the neutral gap excitation of the  $2 + 1/3$  state has magnetorotons similar to those in its cousin state in the LLL. The greatly reduced energy is evidence of different interaction physics.

The interpretation of the modes as gapped neutral collective excitations of the FQHE state is strongly supported by their temperature dependence [30]. The RILS intensity of the modes at  $E_g$  are already significantly reduced by raising the temperature from 42 mK over 100 mK to 250 mK, and are absent in the spectra at 600 mK. Similarly, the mode labeled  $E_{DOS}$  gets broadened and greatly reduced in intensity by increasing the temperature [30]. A quantitative mode analysis of the  $E_g$  band at  $2 + 2/5$  and  $2 + 3/8$  is demanding and uncovers only one mode at  $E_g$  that is centered below  $75 \text{ } \mu\text{eV}$  [39]. A more exact analysis is hindered by a combination of ultralow energies, weakness of the modes, and smaller range of resonance enhancement. Temperature dependent measurements reveal that the modes, particularly  $E_g$ , are already significantly reduced by increasing the temperature from 42 mK to 65 mK and are further weakened by raising the temperature to 100 mK. The strong temperature dependence of the modes underlines the fragility of quantum fluids at these filling factors.

The polarization dependence of modes observed in RILS is exemplified by the results at filling factor  $\nu = 2 + 2/5$  shown in Fig. 5. While it is known that excitations with spin reversal are more intense in cross-polarized scattering, in the presence

of an external magnetic field RILS polarization selection rules are relaxed so that spin as well as charge modes are accessible in cross- as well as copolarized scattering [20]. RILS experiments are typically performed in cross-polarization to suppress parasitic light at  $\omega_L$  that would mask the signal of RRS and of low-energy RILS modes as shown for  $\nu = 2 + 2/5$  and  $\nu = 2.38$  in the Supplemental Material [40]. The spectra in Fig. 5 were obtained by careful suppression of parasitic light at  $\omega_L$  to allow quantitative analyses of RILS and RRS spectra in (H,V) and (V,V) geometries. In these results, the lowest-energy mode  $E_g$  and, as expected, the weak mode  $E_s$  are weak in (V,V) spectra. In addition, at  $2 + 2/5$  the RRS signal is much stronger in (H,V) [40]. The gapped modes are absent for both (H,V) and (V,V) scattering geometries at filling factors slightly away from  $\nu = 2 + 2/5$  by  $\Delta\nu = 0.02$ . Simultaneously, the RRS in (H,V) is significantly reduced. It is evident that in nonresonant excitation the intensity at zero energy is higher for (V,V) compared to (H,V) independent from the filling factor due to parasitic intensity at  $\omega_L$ . Both RRS and RILS spectra exhibit a striking polarization dependence only for filling factors linked to an incompressible FQHE state. We ascribe the polarization dependence in inelastic as well as elastic light scattering to anisotropic susceptibilities  $\chi_{\parallel}$  and  $\chi_{\perp}$  parallel and transverse to the in-plane component of the magnetic field  $B_{\parallel}$  as predicted by theory [13]. The filling factor dependence further corroborates the interpretation from transport experiments that nematic FQHE states are stabilized in the SLL at  $\nu = 2 + 1/3$  [9],  $\nu = 5/2$  [12], and  $\nu = 2 + 2/5$  [41]. This interpretation is consistent with the redshift of the spin-wave (SW) energy resulting in a reduced value of the  $g$  factor due to the collapse of full rotational invariance.

To summarize, gapped low-energy modes have been observed in RILS for  $\nu = 2 + 2/5$ ,  $2 + 3/8$ , and  $2 + 1/3$ . Even for the very fragile states at  $2 + 2/5$  and  $2 + 3/8$  three modes are clearly observable and are interpreted as collective spin and charge modes of the FQHE states. This interpretation is corroborated by the clear filling factor and temperature dependence. A detailed mode analysis for  $\nu = 2 + 1/3$  reveals that the neutral gap excitation spectrum exhibits magnetorotons. The greatly reduced energies from those at  $\nu = 1/3$  indicate weaker quasiparticle interactions. Observations from polarization dependent RILS and RRS measurements at  $\nu = 2 + 2/5$  can be explained by an anisotropic susceptibility consistent with the existence of nematic FQHE states in the SLL in the presence of an in-plane magnetic field [8,9,12]. The reported results provide in-depth insight into the nature of the fragile and enigmatic FQHE states in the SLL and can facilitate in distinguishing between different theoretical scenarios.

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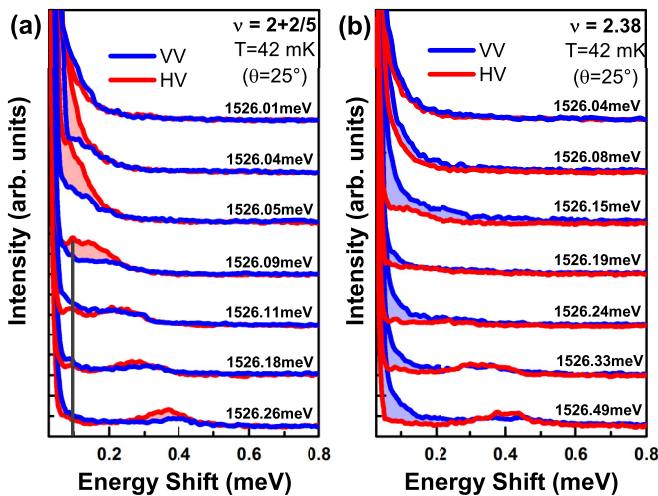


FIG. 5. (Color online) Polarization dependent RILS spectra in (H,V) geometry (red) and (H,H) geometry (blue) for (a)  $\nu = 2 + 2/5$  and (b)  $\nu = 2.38$ , respectively ( $T = 42 \text{ mK}$ ,  $\theta = 25^\circ$ ).

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