

Shubnikov–de Haas oscillations in a two-dimensional electron gas under subterahertz radiation

Q. Shi,¹ P. D. Martin,¹ A. T. Hatke,^{1,2} M. A. Zudov,¹ J. D. Watson,^{3,4} G. C. Gardner,^{4,5} M. J. Manfra,^{3,4,5,6}
L. N. Pfeiffer,⁷ and K. W. West⁷

¹*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

²*National High Magnetic Field Laboratory, Tallahassee, Florida 32310, USA*

³*Department of Physics and Astronomy, Purdue University, West Lafayette, Indiana 47907, USA*

⁴*Birck Nanotechnology Center, Purdue University, West Lafayette, Indiana 47907, USA*

⁵*School of Materials Engineering, Purdue University, West Lafayette, Indiana 47907, USA*

⁶*School of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana 47907, USA*

⁷*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

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We report on magnetotransport measurements in a two-dimensional (2D) electron gas subject to subterahertz radiation in the regime where Shubnikov–de Haas oscillations (SdHOs) and microwave-induced resistance oscillations (MIROs) coexist over a wide magnetic field range, spanning several harmonics of the cyclotron resonance. Surprisingly, we find that the SdHO amplitude is modified by the radiation in a nontrivial way, owing to the oscillatory correction which has the same period and phase as MIROs. This finding challenges our current understanding of microwave photoresistance in 2D electron gas, calling for future investigations.

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When a two-dimensional electron gas (2DEG) is subject to a perpendicular magnetic field B and low temperature T , the longitudinal resistivity ρ exhibits Shubnikov–de Haas oscillations (SdHOs), owing to a quantum correction

$$\delta\rho_{\text{SdH}} = -S \cos \pi \nu, \quad S = 4\rho_0 \lambda \mathcal{D}_T. \quad (1)$$

Here, ν is the filling factor, ρ_0 is the resistivity at $B = 0$, $\lambda = \exp(-\pi/\omega_c \tau_q)$ is the Dingle factor, τ_q is the quantum lifetime, $\mathcal{D}_T = \mathcal{X}_T / \sinh \mathcal{X}_T$, $\mathcal{X}_T = 2\pi^2 k_B T / \hbar \omega_c$, $\omega_c = eB/m^*$ is the cyclotron frequency, and m^* is the effective mass. When a 2DEG is subject to radiation of frequency $\omega = 2\pi f$, ρ also reveals microwave-induced resistance oscillations (MIROs) [1–11] which, according to Refs. [12,13], are given by

$$\delta\rho_{\text{MIRO}} = -2\pi \epsilon \rho_0 \mathcal{P} \eta \lambda^2 \sin 2\pi \epsilon, \quad (2)$$

where $\epsilon = \omega/\omega_c$, \mathcal{P} is the dimensionless radiation intensity [13–17], $\eta = \tau/2\tau_* + 2\tau_{\text{in}}/\tau$, τ is the transport lifetime, τ_{in} is the inelastic lifetime, and $\tau_*^{-1} = 3\tau_0^{-1} - 4\tau_1^{-1} + \tau_2^{-1}$ [18]. When the photoresistance $\delta\rho_{\text{MIRO}}$ approaches the dark resistivity ρ by absolute value, the MIRO minima evolve into zero-resistance states [19–27], which are understood in terms of current domains [28–31].

The majority of MIRO studies have been performed at relatively high T and low f , at which SdHOs are strongly suppressed. Extending experiments to higher f [23,32–39] and lower T yields a regime where SdHOs and MIROs coexist, allowing one to explore possible mixing between these two types of quantum oscillations and to investigate the effect of radiation on SdHOs in general.

It has been known for some time that microwaves suppress SdHOs in the vicinity of cyclotron resonance, $\epsilon \approx 1$ [32,33,35]. As SdHOs are sensitive to the thermal smearing of the Fermi surface, this suppression can be directly linked to absorption, which is indeed the strongest near the cyclotron resonance [40–42]. Away from the cyclotron resonance, our understanding of how microwaves affect SdHOs is definitely

lacking. Some experiments have shown that the effect of microwaves on the SdHO is the weakest near half integer ϵ , which was attributed to the suppression of both inter- and intra-Landau level absorption [34,35]. Another experiment [37] found that as the MIRO minima approach zero, the SdHO amplitude vanishes in proportion with the background resistivity. Reference [37] then argued that in an irradiated 2DEG, ρ_0 in Eq. (1) should be replaced by $\rho_{\text{MIRO}} \approx \rho_0 + \delta\rho_{\text{MIRO}}$.

There exist several mechanisms that could lead to the modification of the SdHO by radiation. First, the absorption coefficient \mathcal{A} is expected to acquire an oscillatory quantum correction [6,40,43–46] which, according to Refs. [40,46], is given by

$$\delta\mathcal{A}_q \simeq 2\mathcal{A}_D \lambda^2 \cos 2\pi \epsilon, \quad (3)$$

where \mathcal{A}_D is a classical absorption described by a Drude formula [16,46,47]. Since oscillations in \mathcal{A} translate to oscillations in T [41,42], Eq. (3) suggests that the microwave-induced suppression of SdHO is maximized near the cyclotron resonance and, to a much lesser extent, near its harmonics,

$$\epsilon = n = 1, 2, 3, \dots \quad (4)$$

In addition, theory also predicts a radiation-induced oscillatory correction, of the order $O(\lambda)$ [48], to the dc resistivity. While the inelastic mechanism produces no such contribution [49], the displacement mechanism dictates that S in Eq. (1) acquires an oscillatory correction and should be replaced by [49,50]

$$S_\omega = S \left[1 - \mathcal{P} \frac{\tau}{\tau_*} \sin^2(\pi \epsilon) \right], \quad (5)$$

suggesting that the SdHO amplitude is minimized at

$$\epsilon = n + 1/2 = 3/2, 5/2, 7/2, \dots, \quad (6)$$

a condition orthogonal to Eq. (4). Finally, the same condition, Eq. (6), can be expected from classical oscillations in magnetoabsorption [51], $\delta\mathcal{A}_c/\mathcal{A}_D \sim -\cos 2\pi \epsilon$, which can be stronger than quantum oscillations, given by Eq. (3), in a typical 2DEG.

In this Rapid Communication we experimentally investigate the photoresistance in high-quality 2DEGs. Using high f , low \mathcal{P} , and low T allows us to overlap MIROs and SdHOs over a wide range of ϵ and to investigate the SdHO wave form near multiple harmonics of the cyclotron resonance. Our data reveal pronounced modulation of the SdHO amplitude which persists to $\epsilon \approx 6$. Surprisingly, even though the modulation is periodic in ϵ , it cannot be described by either Eqs. (3) and (4) or Eqs. (5) and (6). Instead, the radiation-modified SdHO amplitude closely replicates the MIRO wave form [see Eq. (2)], suggesting a nontrivial mixing of MIROs and SdHOs. While it is well established that quantum oscillations of the order $O(\lambda^2)$ interfere with each other [14,15,52–58], the observed correlation between MIRO $\sim O(\lambda^2)$ and SdHO $\sim O(\lambda)$ is totally unexpected [59].

While we have obtained similar findings from a variety of samples grown at Princeton and Purdue, in what follows we present the results from two Purdue-grown Hall bars, I and II, of width 300 μm and 200 μm , respectively. Sample I is fabricated from a 30-nm-wide $\text{Al}_{0.0015}\text{Ga}_{0.9985}\text{As}/\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$ quantum well, with density $n_e \approx 3.1 \times 10^{11} \text{ cm}^{-2}$ and mobility $\mu \approx 3.6 \times 10^6 \text{ cm}^2/\text{V s}$. Sample II contains a 30-nm-wide $\text{GaAs}/\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$ quantum well, with $n_e \approx 2.6 \times 10^{11} \text{ cm}^{-2}$ and $\mu \approx 2.1 \times 10^7 \text{ cm}^2/\text{V s}$. The resistivity was measured using a standard low-frequency lock-in technique, in sweeping B , with f from 0.2 to 0.4 THz, generated by backward wave oscillators.

To facilitate the discussion of our results, we first define the relevant quantities and introduce convenient notations. In the absence of microwave radiation, the resistivity can be represented as

$$\rho = \rho_{\text{sm}} + \delta\rho_{\text{SdH}}, \quad (7)$$

where ρ_{sm} is the smooth part of the resistivity [60] and $\delta\rho_{\text{SdH}}$ is given by Eq. (1). When the radiation is present, we write the resistivity as

$$\rho_\omega = \rho_{\text{MIRO}} + \delta\rho_{\text{SdH}\omega}, \quad (8)$$

where we have introduced $\rho_{\text{MIRO}} = \rho_{\text{sm}\omega} + \delta\rho_{\text{MIRO}}$ containing slowly varying background $\rho_{\text{sm}\omega}$ [61] and MIRO photoresistance [see Eq. (2)]. The main goal of our study is to examine if and how $\delta\rho_{\text{SdH}\omega}$ is different from $\delta\rho_{\text{SdH}}$.

In Fig. 1(a) [Fig. 1(b)] we present the magnetoresistivity $\rho_\omega(B)$ measured in sample I (II) irradiated by microwaves of $f = 378$ (290) GHz at $T = 0.3$ K. The vertical lines are drawn at the cyclotron resonance harmonics, $\epsilon = \omega/\omega_c = 2, 3, 4, \dots$. In both samples, the data reveal pronounced MIROs, persisting down to $B \approx 0.05$ T, and SdHOs, terminating around $B \approx 0.15$ T. Owing to high f and low T , there exists a wide range of B where SdHOs and MIROs coexist. Most importantly, this range extends over several harmonics of the cyclotron resonance, spanning up to $\epsilon \approx 6$ and $\epsilon \approx 5$ for samples I and II, respectively. We immediately notice that, under irradiation, the SdHO amplitude S_ω is *not* a monotonic function of B , in contrast to the “dark” amplitude S described by Eq. (1). We thus conclude that the effect of radiation on SdHO is not limited to nonresonant heating.

To get more insight into the radiation-induced changes of SdHOs, one needs to separate ρ_{MIRO} and $\delta\rho_{\text{SdH}\omega}$, entering

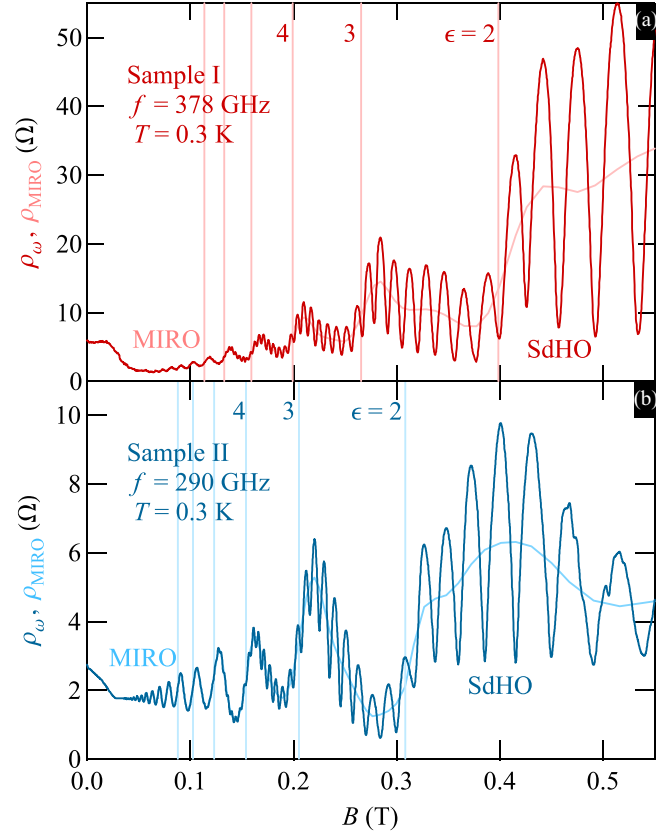


FIG. 1. (Color online) (a) [(b)] Magnetoresistivity $\rho_\omega(B)$ (dark curves) measured in sample I (II) irradiated by microwaves of $f = 378$ GHz ($f = 290$ GHz) at $T = 0.3$ K. Vertical lines are drawn at the cyclotron resonance harmonics, $\epsilon = \omega/\omega_c = 2, 3, 4, \dots$. Both panels also show $\rho_{\text{MIRO}}(B)$ (light curves) obtained by averaging out SdHOs [see Eq. (8)].

Eq. (8). Since ρ_{MIRO} oscillate much slower than SdHOs [cf. Eq. (7)], it can be easily obtained by averaging out faster SdHOs. Obtained in this way, $\rho_{\text{MIRO}}(B)$ is shown in both panels of Fig. 1 by light curves running midway between the SdHO maxima and minima.

Having found ρ_{MIRO} , we now use Eq. (8) to obtain $\delta\rho_{\text{SdH}\omega}$ by subtracting ρ_{MIRO} from ρ_ω , both shown in Fig. 1. The results for samples I and II are presented as a function of ϵ in Figs. 2(a) and 2(b), respectively. For comparison, we also include ρ_{MIRO} and $\delta\rho_{\text{SdH}}$, as marked. The latter was found using Eq. (7) by subtracting the smooth part of the resistivity ρ_{sm} from $\rho(B)$ measured without irradiation. Direct examination of the SdHO reveals that $|\delta\rho_{\text{SdH}\omega}| \leq |\delta\rho_{\text{SdH}}|$ in the entire range of ϵ , which is expected because the radiation elevates the temperature. In addition, one can now clearly see that, in contrast to $\delta\rho_{\text{SdH}}$, which monotonically decays with ϵ in accordance with Eq. (1), $\delta\rho_{\text{SdH}\omega}$ exhibits clear signs of modulation with the period close to unity. Furthermore, a comparison of $\delta\rho_{\text{SdH}\omega}$ and ρ_{MIRO} hints at the strong correlation between the two quantities.

We next extract the amplitude of $\delta\rho_{\text{SdH}\omega}$, shown in Fig. 2, and examine it in more detail. In Fig. 3 we present the extracted amplitude S_ω (open circles) and ρ_{MIRO} (solid circles) as a function of ϵ on a log-linear scale. Once plotted together, the

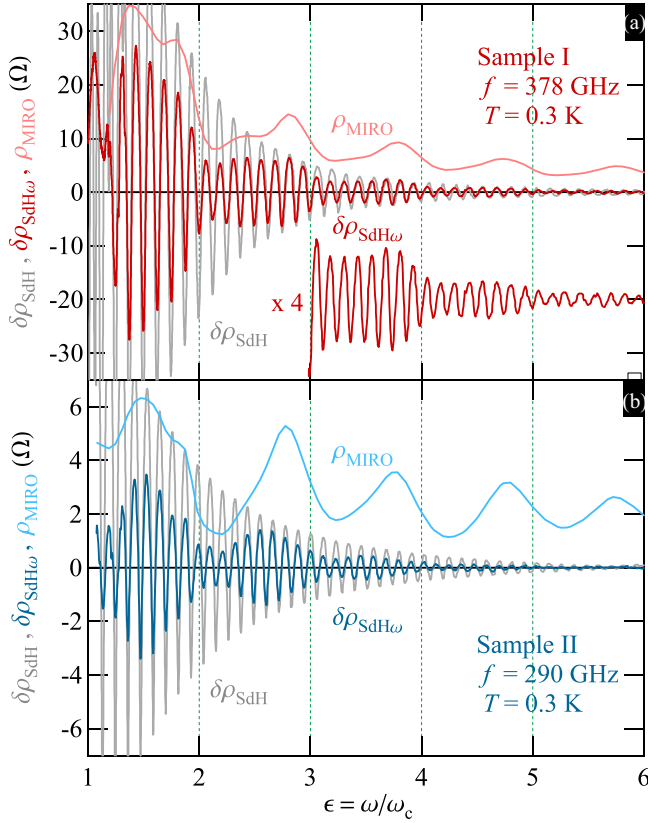


FIG. 2. (Color online) (a) [(b)] $\delta\rho_{\text{SdH}}$, $\delta\rho_{\text{SdH}\omega}$, and ρ_{MIRO} , obtained from the data in Fig. 1(a) [Fig. 1(b)], vs ϵ . (a) also shows $\delta\rho_{\text{SdH}\omega}$, multiplied by 4 and offset by 20 Ω .

correlation between S_ω and ρ_{MIRO} becomes very clear—both quantities oscillate in phase with each other. In other words, radiation induces minima in the SdHO amplitude at

$$\epsilon \approx n + 1/4 = 5/4, 9/4, 13/4, \dots, \quad (9)$$

in contrast to both the scenario considering oscillations in magnetoabsorption [Eq. (3)] and the one predicting direct modification of the SdHO [Eq. (5)].

In the remaining part of this Rapid Communication we search for an empirical relation describing the SdHO amplitude in the presence of radiation. To this end, we extract and compare the oscillatory parts in S_ω and in ρ_{MIRO} . More specifically, we introduce the dimensionless quantity $\delta S_\omega/S = S_\omega/S - 1$, where S is the smooth, nonoscillating part of the SdHO amplitude shown in Fig. 3 by straight lines, and present the results in Fig. 4. For comparison, we also plot the oscillatory part of MIRO, $\delta\rho_{\text{MIRO}}/(2\pi\epsilon\lambda^2\rho_0)$.

We immediately see that both quantities oscillate around zero without noticeable decay, confirming that exponential factors have been properly eliminated. As already anticipated, a very good agreement in both the period and the phase is found for the whole range of ϵ studied. This finding indicates that observed oscillations in the SdHO amplitude are closely related to MIROs and thus are unlikely to originate from resonant heating caused by oscillations in magnetoabsorption [Eq. (3)]. We thus conclude that under the presence of radiation

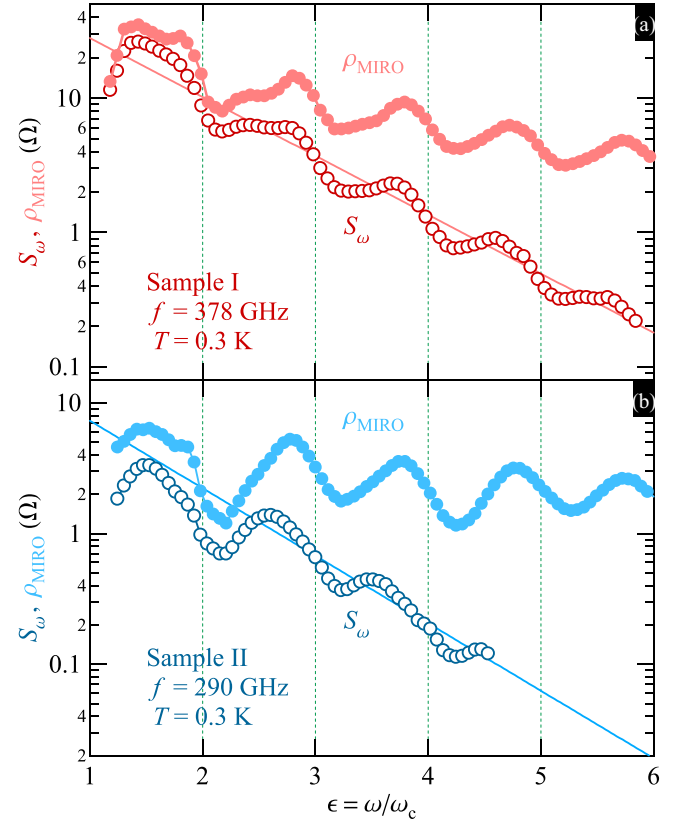


FIG. 3. (Color online) (a) [(b)] ρ_{MIRO} (solid circles) and S_ω (open circles), extracted from the data in Fig. 1(a) [Fig. 1(b)], as a function of ϵ .

the SdHO amplitude is given by

$$S_\omega \approx S(1 - \alpha \sin 2\pi\epsilon), \quad (10)$$

where α is a dimensionless ϵ -independent constant, showing that δS_ω is a correction of order $O(\lambda)$, just as the SdHOs themselves.

The most surprising finding, however, is a *quantitative* correlation between δS_ω and $\delta\rho_{\text{MIRO}}$, namely,

$$\frac{\delta S_\omega}{S} \approx \frac{\delta\rho_{\text{MIRO}}}{2\pi\epsilon\lambda^2\rho_0} \approx -\alpha \sin 2\pi\epsilon, \quad (11)$$

where all parameters have been obtained experimentally. The observed correlation is completely unexpected and is found almost everywhere, except at $\epsilon \lesssim 2$ in sample II. The latter can be linked to increased absorption close to the cyclotron resonance, where SdHOs are suppressed due to resonant heating [32,33,35,62]. The absence of such a deviation in sample I can be attributed to considerably higher f , which reduces the influence of the cyclotron absorption peak. Interestingly, combining Eq. (11) with Eq. (2), one finds that $\alpha \approx \mathcal{P}\eta$.

Finally, we examine the proposal of Ref. [37] that the SdHO under irradiation can be described by Eq. (1) with ρ_0 replaced by $\rho_0 + \delta\rho_{\text{MIRO}}$ [63]. Taking this approach, one obtains $\delta S_\omega/S = \delta\rho_{\text{MIRO}}/\rho_0$, a result similar to Eq. (11), but with an extra factor $2\pi\epsilon\lambda^2$, which has significant dependence on ϵ . Indeed, as ϵ increases from 2 to 6, $2\pi\epsilon\lambda^2$ decreases by nearly a factor of 5 for sample I. In contrast, our data shown

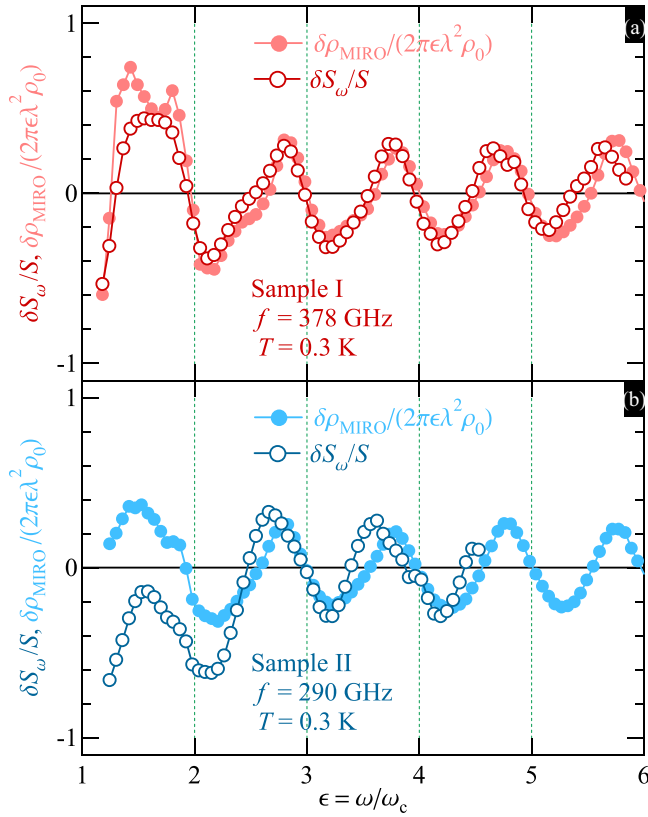


FIG. 4. (Color online) (a) [(b)] $\delta S_\omega/S$ (open circles) and $\delta\rho_{\text{MIRO}}/(2\pi\epsilon\lambda^2\rho_0)$ (solid circles), extracted from the data in Fig. 1(a) [Fig. 1(b)], as a function of ϵ .

in Fig. 4(a) show virtually no decay at $\epsilon \gtrsim 2$. In addition, if this factor were actually present, the correction to SdHO would

have been up to >3 (>5) times larger than observed in sample I (sample II). We thus conclude that the proposal of Ref. [37] is irrelevant to our findings.

In summary, we have studied the photoresistance in high-quality 2DEG subject to low temperatures and high microwave frequencies, which allowed us to overlap MIRO and SdHO over multiple harmonics of the cyclotron resonance. Our data revealed pronounced modulation of the SdHO which is periodic in ϵ , with the period equal to unity, and the phase matching that of the MIRO. This result does not fit existing theories considering either magnetoabsorption or photoresistance. Most remarkably, we have found that once the exponential factors are eliminated, the oscillatory part of the SdHO amplitude matches that of MIRO *quantitatively*, without any adjustable parameters. This finding allowed us to deduce an empirical relation for the SdHO amplitude in irradiated 2DEG, given by Eqs. (10) and (11). Taken together, our study reveals that the current understanding of SdHOs in irradiated 2DEG is lacking, calling for further investigations.

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- [59] Under conditions of our experiment, in sample I (II) we estimate τ_q to be 2 (3) ps from SdHOs and 4.4 (8.2) ps from MIROs. However, the accurate determination of SdHO τ_q requires extremely slow sweeps at very low B , which were not attempted because the exact knowledge of τ_q is not crucial for our study.
- [60] ρ_{sm} is not significantly different from ρ_0 , except in 2DEG exhibiting strong negative magnetoresistance [64–67].
- [61] Strictly speaking, $\rho_{sm\omega} \neq \rho_{sm}$ because of radiation-induced heating and T dependence of ρ_{sm} .
- [62] While all samples surveyed have shown a correlation between MIROs and SdHOs, in many of them MIROs rapidly decayed with increasing f . As a result, modulation of the SdHOs was observed only up to $\epsilon \approx 2$, where the excess heating due to the proximity to the cyclotron resonance precluded a quantitative analysis of the wave forms.
- [63] We are not suggesting that this scenario is justified.
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