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Optical determination of the spin polarization of a quantum Hall ferromagnet

M.J. Manfra [a], B.B. Goldberg [a,*], L. Pfeiffer [b], K. West [b]

[a] *Department of Physics, Boston University, Boston, MA 02215, USA*
[b] *Bell Laboratories, Lucent Technologies, Murray Hill, NJ 07974, USA*

Abstract

Recent experimental and theoretical investigations have resulted in a shift in our understanding of the $\nu = 1$ quantum Hall state. There now exists a wealth of evidence that the excitation gap and the resulting quasi-particle spectrum at $\nu = 1$ are due *predominately* to the ferromagnetic many-body exchange interaction. A great variety of experimentally observed correlations at $\nu = 1$ *cannot* be incorporated into a perturbative expansion around the single-particle model, a scheme long thought to describe the integral quantum Hall effect (IQHE) at filling factor 1. Theorists now refer to the $\nu = 1$ state as the quantum Hall ferromagnet. In this paper, we review recent theoretical and experimental progress and detail our own optical investigations of the $\nu = 1$ quantum Hall regime.

Keywords: Quantum Hall effect; Skyrmions; Magneto-optics

1. Introduction

Some 15 years after the discovery of the fractional quantum Hall effect [1], the study of two-dimensional electron systems (2DES) confined to the lowest Landau level (LLL) continues to be a fertile laboratory for the investigation of many-body interactions. The proliferation of experimentally observed fractional Hall states [2], the discovery of composite fermions at half-filling [3], and the possibility for exotic spin-unpolarized fractional ground states [4] are but a few examples of the anomalies that continue to challenge our understanding of strong electron–electron correlations in the extreme magnetic quantum limit. Recently, the $\nu = 1$ quantum Hall state, a region thought to be well-understood within the single-particle model of the integral quantum Hall effect (IQHE), has been added to the catalog of states whose physics is dominated by many-body interactions.

The importance of the many-body exchange coupling for determining the excitation spectrum at $\nu = 1$ can be seen in a fairly intuitive manner by looking at the relevant energy scales. Filling factor $\nu = 1$ corresponds to having exactly one electron for each available state in the lowest-energy spin-up Landau level. Given the large magnetic fields typically necessary to observe the $\nu = 1$ quantum Hall state ($5\,\mathrm{T} < B < 10\,\mathrm{T}$), it may at first appear odd to call this state the quantum Hall ferromagnet. Ferromagnetism is usually associated with the spontaneous alignment of spins in the absence of an external field.

* Corresponding author. E-mail: goldberg@bu.edu.

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But at $\nu = 1$ all the electrons share a common quantized kinetic energy which is conventionally chosen as the zero of energy. The gap to the next orbital Landau level is typically of the order of 200 K. Thus, at low temperatures the electron's kinetic energy is essentially frozen out and does not affect the system's dynamics. The two remaining energy scales are the single-particle Zeeman energy and many-body exchange. In GaAs systems the Zeeman splitting is only of order 2 K while the typical electron–electron interaction scale is about 100 K. A single spin reversal, which is the lowest-energy single-particle excitation, costs a great deal in ferromagnetic exchange energy which is lowered by keeping neighboring spins aligned. Thus, it may be that the physics of electrons interacting within a single-orbital Landau level with such a small Zeeman coupling may not be appreciably different from that of a ferromagnet in the presence of a small symmetry-breaking field. This is the essence of most recent theoretical approaches to the quantum Hall effect at $\nu = 1$.

The notion of the quantum Hall ferromagnet was first advanced in the seminal work of Sondhi [5]. His approach incorporated the use of the nonlinear σ model (NLσ) of isotropic ferromagnets in the study of the $\nu = 1$ quantum Hall effect. The NLσ model has been extensively studied and its elementary excitations are known to possess highly nontrivial long-range spin order. These quasi-particles are called Skyrmions and consist of a radial spin density that is reversed at the center but gradually heals to the ferromagnetic background over many magnetic lengths. Generally, real quantum Hall ferromagnets possess a small, but finite, Zeeman coupling which must be included for realistic calculations of energetics and spin polarization. An extended Hartree–Fock theory of quantum Hall Skyrmions was developed by Fertig and coworkers [6] and provides a quantitative basis for comparison with experiment. The energetics and spin of real quantum Hall Skyrmions, or charged spin texture excitations (CSTEs), are largely determined by the parameter $\tilde{g} = g\mu_B B/(e^2/\varepsilon_0 l_0)$, the ratio of the Zeeman energy to the Coulomb interaction scale. The Zeeman term favors small Skyrmions while the Coulomb term tries to maximize the size of the spin texture. For typical samples under investigation $\tilde{g} \sim 0.015$. For this value of $\tilde{g}$ the CSTE is always lower in energy than the single spin-flip excitation and involves the reversal of approximately 3 spins. It is the excitation of Skyrmions that is responsible for the rapid suppression of the spin polarization for excursions away from $\nu = 1$.

2. Optical detection of Skyrmions

Our determination of the spin polarization of the $\nu = 1$ state is based on polarization-resolved magneto-absorption spectroscopy. The 2DES studied in these experiments resides at the interface of a single-side n-modulation-doped AlGaAs–GaAs single quantum well (SQW). We are reporting the results of measurements made on three distinct wafers. All GaAs wells are 250 Å wide. For sample A, the electron density is $N_s = 1.8 \times 10^{11}\ \text{cm}^{-2}$ with a mobility $\mu = 2.6 \times 10^6\ \text{cm}^2/\text{V s}$. Sample B has a density of $N_s = 1.2 \times 10^{11}\ \text{cm}^{-2}$ and a mobility $\mu = 2.0 \times 10^6\ \text{cm}^2/\text{V s}$. For sample C, $N_s = 1.7 \times 10^{11}\ \text{cm}^{-2}$ and $\mu = 3.6 \times 10^6\ \text{cm}^2/\text{V s}$. In order to perform absorption measurements the samples were mounted strain-free and thinned to $\sim 0.5\ \mu\text{m}$ with a jet-etching technique. Collimated light centered at 810 nm is passed to, and collected from, the sample through a fiber optic system that fits in a He^3 refrigerator mounted in a 12 T magnet. Polarization analysis is done in situ with a circular analyzer placed immediately following the sample. Reversing the direction of the magnetic field relative to the direction of propagation of light allows for the independent monitoring of the LCP and RCP spectra.

An example of the absorption spectra of sample A as a function of magnetic field in each polarization is shown in Fig. 1. As the Fermi level moves through $\nu = 1$ the absorption to the lower-energy spin-up state quenches, concomitant with a peaking of the absorption into the higher-energy spin-down state. This is clearly seen in the inset of Fig. 1 where the absorption coefficient $\alpha = -1/L_w \ln(I(B)/I(0))$ (L_w is the quantum well width and I the measured transmission intensity) is plotted as a function of B in the neighborhood of $\nu = 1$ for each polarization. These data indicate the rapidly changing populations of the two spin states on both the high- and low-field sides of $\nu = 1$. It is clear that this behavior is quite robust: it is seen in all three samples as the Fermi level is swept through $\nu = 1$.

Our determination of the spin polarization from the absorption spectra follows from Fermi's golden rule,

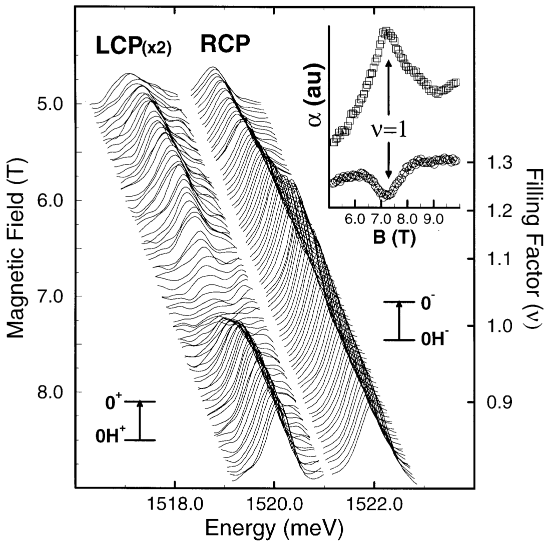

Fig. 1. Absorption spectra in LCP and RCP as a function of magnetic field at $T = 1.5$ K for sample A. The RCP spectra are offset by +1 meV in energy for clarity. The main optical transitions to the two lowest electron spin states are shown alongside the spectra. The inset displays the peak absorption coefficient α in the vicinity of $\nu = 1$.

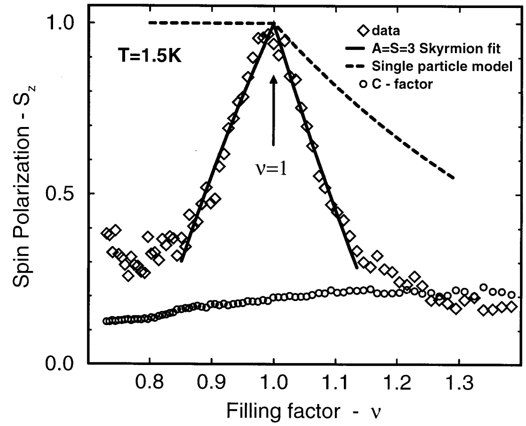

Fig. 2. Spin polarization of sample A plotted versus filling factor and compared with both a single particle and Skyrmion-based model. The single-particle polarization is based on a simple counting argument, one spin flip per unpaired flux quantum for $\nu > 1$ and $S_z = 1$ for $\nu < 1$. The Skyrmion model has been detailed elsewhere [7] and fits well for 3 spin-flips per unpaired flux quanta. The magnetic field dependence of the constant of proportionality C is also shown.

which relates the integrated absorption intensity to the available density of states in each spin band, and on a simple sum rule which enforces particle conservation, $N_\uparrow + N_\downarrow = N$ where $N_{\uparrow(\downarrow)}$ is the number of spin-up (-down) electrons in the lowest Landau level. The sum rule constrains the total available density of states at any given magnetic field. We proceed with the calculation of S_z by first dividing the integrated peak absorption in each polarization by the calculated optical matrix elements. The resulting quantity is proportional to the available density of states

$$\frac{I_{ij}}{f_{ij}} = CN_{A_j}, \tag{1}$$

where I_{ij} is the integrated absorption, f_{ij} is the optical matrix element and C is the constant of proportionality which accounts for all unmeasured experimental couplings – optical collection efficiency, detector efficiency, etc. – which otherwise would be needed to relate the measured integrated intensity to the available density of states. Since the left- and right-circularly polarized spectra provide two independent equations – one for each spin band, but with the same experimental coupling C – the additional constraint enforced by Eq. (2) allows for determination of $N_{A_{\uparrow(\downarrow)}}$ and the elimination of the constant of proportionality C. Finally, the spin polarization per particle is

$$S_z = \frac{N_\uparrow - N_\downarrow}{N} = \frac{N_{A_\downarrow} - N_{A_\uparrow}}{N}. \tag{2}$$

Thus, this experiment allows for the determination of the absolute spin polarization, free of any fitting parameters.

Fig. 2 plots the spin polarization versus filling factor for sample A determined from the data in Fig. 1 and compares it with both single particle and Skyrmion-based models. Previous calculations have shown that a single-particle model based on the exchange-enhanced g-factor that modulates the overlap of the two electron spin levels fails to capture the behavior of S_z, especially for $\nu < 1$ [7]. On the other hand, the data conform well to the Skyrmion-based model. In this scheme the number of reversed spins is quantized [8] so that the component of total spin along the direction of the Zeeman field is given by

$$S_z = N/2 - (A+1)|N - N_\phi| \tag{3}$$

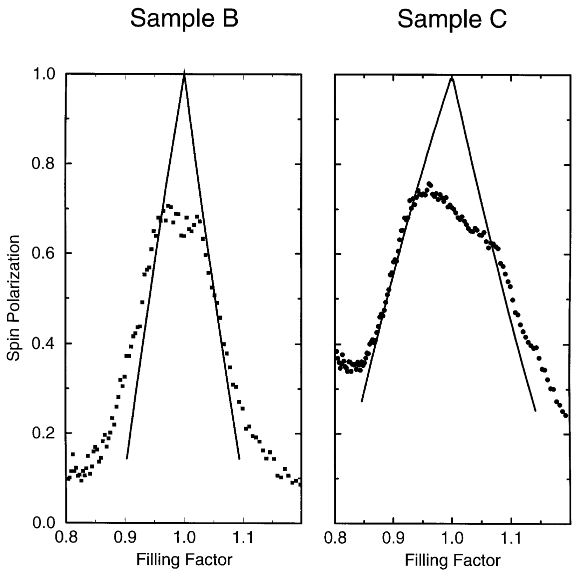

Fig. 3. Spin polarization of sample B and sample C plotted versus filling factor. The lines drawn show the expected polarization for a Skyrmion model with 5 (3) spin-flips per excitation, respectively.

for $N > N_\phi$, where N the number of electrons in the system, N_ϕ is the number of states available and $|N - N_\phi|$ is the number of Skyrmions present. For $N < N_\phi$, S_z is given by

$$S_z = N/2 - (A)|N - N_\phi|, \tag{4}$$

A is an integer quantum number which depends on the relative strength of the Zeeman and Coulomb interaction terms and is the relevant measure of the Skyrmion size [8]. For samples with $\nu = 1$ at 7 T, Hartree–Fock calculations [6] predict A should be close to 3, indicating each excitation induces 3 spin flips. Sample A, with $\nu = 1$ at 7.2 T, is in agreement with this prediction.

The measured spin polarizations for sample B and C are shown in Fig. 3. Again, in each of these samples we see a rapid quenching of the degree of polarization for excursions away from $\nu = 1$. In both these samples, however, the polarization does not approach unity exactly at $\nu = 1$. Despite the seemingly similar behavior, we, in fact, believe that different mechanisms are involved for each sample. We begin the discussion with sample B. For sample B $\nu = 1$ occurs at 4.75 T. The solid line drawn corresponds to $A = 5$, implying 5 spin flips per excitation. This result is expected since at smaller magnetic field $\tilde{g}$ is smaller resulting in a larger effective spin texture. What distinguishes sample B is its very pronounced temperature dependence below $T = 1$ K. By a temperature of $T = 4.2$ K all evidence of $\nu = 1$ have disappeared from the optical spectra. The polarization at $\nu = 1$ is still increasing for decreasing temperature at $T = 0.5$ K, the lowest temperature obtainable in this experimental configuration. These findings are consistent with the material parameters of sample B. Its lower density results in smaller intrinsic gaps at $\nu = 1$ and its higher degree of disorder as manifest in a slightly lower mobility both point to a thermal suppression of the $\nu = 1$ polarization. A somewhat different behavior is seen in sample C. Sample C has $\nu = 1$ at 7 T. For deviations away from $\nu = 1$ sample C is also well fit by a theory with 3 spin flips per excitation, a result consistent with the behavior observed in sample A. Yet like sample B, sample C's spin polarization does not reach unity at $\nu = 1$. One might suspect that the same mechanism is at work here as in sample B. However, unlike for sample B, below $T = 1$ K the $\nu = 1$ polarization of sample C is largely independent of temperature. The peaking of the RCP absorption coefficient has essentially saturated. Additionally, in transport this sample exhibits a relatively wide quantum Hall plateau, suggesting that localization associated with reduced screening of impurity potentials in the incompressible $\nu = 1$ quantum Hall state may be having a significant impact on the physics of this sample. Further indication of the effects of localization is given by the slightly broader and asymmetric absorption peaks observed in this sample. The allowed absorption transitions outside the defined region of energy integration would naturally lead to the observed saturation in polarization. It must be stressed, however, that the dominant behavior observed in all samples is completely consistent with the Skyrmion model: the system depolarizes on both sides of $\nu = 1$ and the excitations involve a large number of spin flips.

2.1. *Temperature dependence of spin polarization at* $\nu = 1$

The discussion so far has focussed on the Skyrmionic excitations of the quantum Hall ferromagnet. Skyrmions are the *charged* excitations of the system and are driven by adding or removing

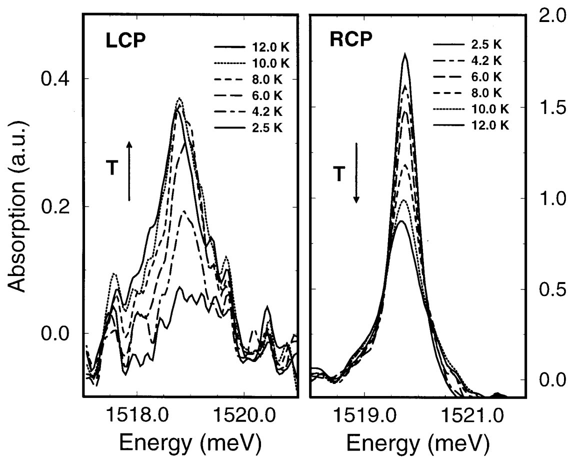

Fig. 4. Absorption spectra taken in LCP and RCP at $\nu = 1$ as a function of temperature for sample A. The arrows point in the direction of spectral change for increasing temperature.

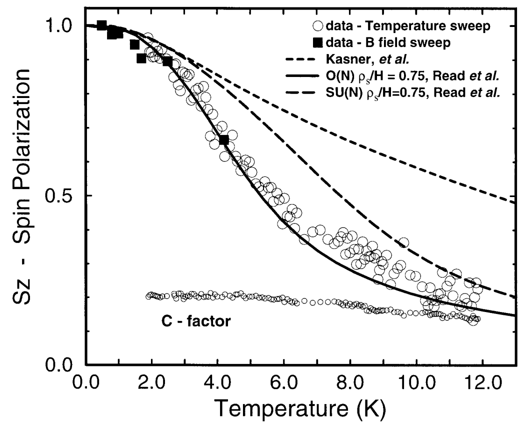

Fig. 5. Temperature dependence of the spin polarization at $\nu = 1$ as determined by polarized absorption spectroscopy. The theoretical curves from the CQFM of Read and Sachdev [11] the many-body perturbation curve of Kasner and MacDonald [12] are also displayed.

charge, or equivalently, adding or removing magnetic flux. The charged nature of Skyrmions is evidenced by the sensitivity of transport measurements to their presence [9, 10]. But Skyrmions are not the only excitations of the quantum Hall ferromagnet. As with any highly correlated spin system, we expect a branch of low-lying neutral spin-wave excitations which may affect the spin polarization even at $\nu = 1$ for finite temperatures. In order to explore the nature of the neutral excitations of the quantum Hall ferromagnet we have completed a detailed study of the thermal behavior of the spin polarization exactly at $\nu = 1$.

Band-gap absorption spectra show striking temperature dependence due to changes in the occupations of the spin-split states of the ground Landau level at filling factors near $\nu = 1$. The temperature-dependent occupations of the spin-up and spin-down electron levels leads to a thermodynamic measure of the spin polarization. Fig. 4 displays the temperature dependence of the absorption taken in LCP and RCP at $\nu = 1$ for sample A. The decrease in absorption for decreasing temperature into the lower-energy spin state in LCP is correlated to the increase in absorption into the higher-energy spin state in RCP. As the temperature decreases, there are fewer available states for optical transitions in the lower-energy spin component and more available in the higher-energy spin component. This data is convincing evidence that the temperature-dependent absorption monitors the Fermi distribution of a two-level system. S_z versus T is plotted from 500 mK to 12 K in Fig. 5. It is important to note that C is independent of temperature (see Fig. 5), giving further indication that only the occupancy of the two levels is changing over this temperature range. Additionally, we display several values of $S_z(T)$ obtained independently by sweeping magnetic field at constant temperature. The consistency between the spin-polarization determined from magnetic field sweeps and the spin-polarization determined from temperature sweeps is quite good.

The rapid quenching of the $\nu = 1$ polarization with increasing temperature is further evidence for the highly correlated nature of the quantum Hall ferromagnet. Any theoretical approach to its thermodynamics must account for the collective-magnetization excitations of the ferromagnetic ground state which are expected to dominate the finite-temperature spin polarization. Initial theoretical work [13] which included independent spin-waves displays a much weaker temperature dependence of the spin polarization than our data indicate. Kasner and MacDonald [12] have incorporated spin-wave excitations into a many-body perturbation theory through the inclusion of a self-energy insertion consisting of a ladder sum of repeated interactions between HF electrons of one spin and holes of the opposite spin. Their theoretical S_z versus T curve appropriate to our experimental conditions, including finite well thickness effects, is shown in Fig. 5. The low-temperature reduction of $S_z(T)$ is dominated by the long-wavelength spin-wave contribution and compares favorably with the data. At higher temperatures, however, agreement is less convincing. This perturbative approach is limited by its inability to consistently account for spin-wave–spin-wave interactions.

Read and Sachdev have developed a continuum quantum field theory of systems with a ferromagnetic ground state that is applicable to the $\nu = 1$ quantum Hall state [11]. The description is analogous to an insulating quantum Heisenberg ferromagnet, and the low-temperature behavior of the two systems is expected to be similar. The continuum quantum ferromagnet (CQFM) contains a conserved topological current representing the number density and current of Skyrmions. Most importantly to our discussion of the $\nu = 1$ spin thermodynamics, the finite temperature CQFM systematically accounts for spin-wave–spin-wave interactions which dominate the spin thermodynamics in the regime $k_B T > H$, where $H = g\mu_B B$ is the Zeeman energy. The only parameter in the system is set by the ratio of the energy scales ρ_s, the ferromagnetic spin stiffness, and H. In the limit of zero well thickness, $\rho_s = e^2/(16\sqrt{2\pi}\varepsilon l_B)$. The ratio calculated for our GaAs SQW, including the effects of finite well thickness, is $\rho_s/H \sim 0.77$ [14]. Scaling functions for the spin polarization can be generated in the large N limit when the symmetry group O(3) is generalized to O(N) or SU(N). The large N expansion is not a perturbative expansion in the strength of the interactions but rather a saddle-point expansion which preserves the symmetry and couplings of the underlying Hamiltonian. Thus, evaluating the spin polarization for both O(N) and SU(N) in the $N \to \infty$ limit does not correspond to choosing different symmetry groups for the physical system since $O(3) \cong SU(2)$. It will, however, alter the resulting form of the scaling functions at the mean-field level. In Fig. 5 the O(N) and SU(N) limits of the CQFM with $\rho_s/H = 0.75$ are displayed. We find excellent agreement over the entire range of measured temperatures. It is important to note that there are no free parameters in this comparison of data and theory. All couplings are known and are specific to our system. It is clear that the physics of collective-magnetization excitations captured by the CQFM is crucial to reproducing the observed temperature dependence of S_z.

Recently, Chakraborty et al. have performed exact diagonalization studies of the temperature dependence of the spin-polarization [15]. In general, such finite-size calculations are especially relevant in the regime of larger $\tilde{g}$ where the Hartree–Fock and field theoretic approaches are less accurate. When combined with a softening of the Coulomb interaction due to the extent of the wave function in $\hat{z}$, the calculations show excellent agreement with our experimental data.

3. Emission spectroscopy and the role of the exciton in absorption

From the preceding discussion it is apparent that magneto-absorption spectroscopy is a sensitive probe of the occupancy of the spin-split lowest Landau level. Yet its application to spectroscopy of the quantum Hall regime remains limited to a few studies. By far the most widely used optical probe of the 2DES in the quantum Hall regime has been band-gap emission spectroscopy where the interplay of excitonic final state interactions, QHE localization, and many-body screening are known to make the extraction of level populations quite complicated. In order to clarify the differences between the various optical probes we have very recently undertaken the *simultaneous* measurement of the absorption, PL and PLE spectra from the 2DES in the quantum Hall regime. The full results of this study will be presented in a future publication, but here we will briefly discuss some of the differences between absorption and emission spectroscopy as well as present some of our preliminary low magnetic field emission data. While a full analysis of the PL is not possible at this time it will be clear from the raw spectra the importance of relaxation effects and many-body interactions in the emission process.

Magneto-absorption spectroscopy has several advantages that make it a unique probe of the spin polarization of the 2DES. The integrated absorption intensity can be related, in a very straightforward way, directly to the available density of states in the lowest Landau level. Unlike emission, absorption is largely unaffected by complicated relaxation effects

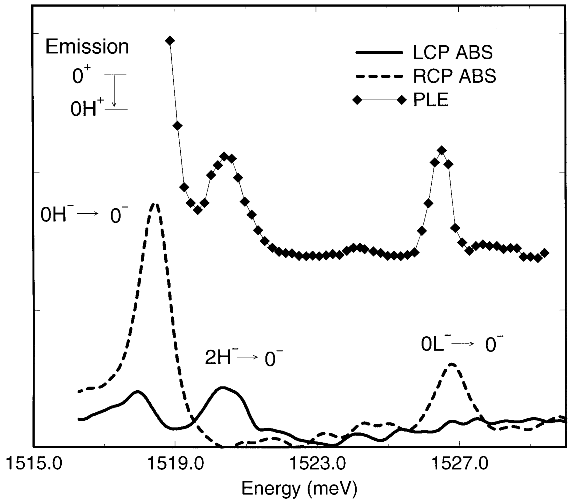

Fig. 6. Integrated ground-state emission intensity at $\nu = 1$ as a function of laser excitation energy. Also displayed are the LCP and RCP direct absorption versus energy at $\nu = 1$. Main absorption transitions are indicated.

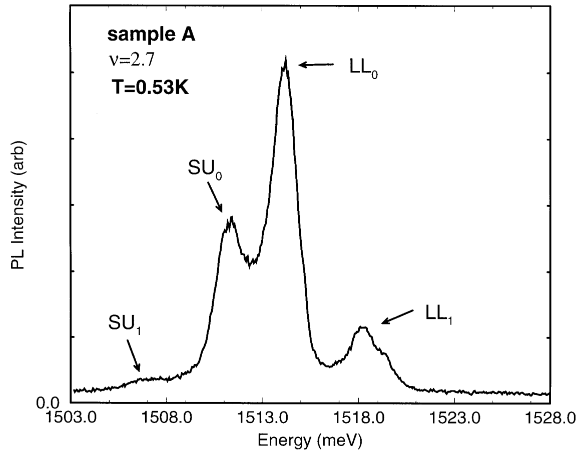

Fig. 7. Photoluminescence spectra taken at $\nu = 2.7$ and $T = 0.53$ K for sample A. In addition to luminescence from the two lowest Landau levels (LL_0 and LL_1), two lower-energy peaks labeled SU_0 and SU_1 are evident.

that may obscure population changes. This is manifest in the strict adherence to the angular momentum selection rules in the absorption process and the good agreement between observed and calculated oscillator strength [7]. Additionally, absorption gives two uncorrelated measurements: the absorption in the LCP channel is *independent* of absorption in the RCP channel so that the relative changes in intensity do, in fact, reflect changes in the population of the two spin bands. In emission, however, the two spin bands are inherently coupled. In the absence of significant non-radiative channels, the minority photo-excited hole must eventually emit a photon on recombination, and hence the emission from the two electron levels must be correlated. Furthermore, our data indicate that relaxation processes have the effect of mixing the hole angular momentum states. In Fig. 6 we plot the response of the ground state emission in LCP at $\nu = 1$ and $T = 0.53$ K for sample A. Also displayed in this figure are the LCP and RCP absorption coefficients taken at the same time. The correspondence between the luminescence and the *combined* absorption coefficient is striking. The emission in LCP increases whenever we have absorption, regardless of the angular momentum of the hole-state created by the absorbed photon. Thus, holes created at absorption must change angular momentum character before recombining.

Some of the complexities due to many-body interactions in PL spectroscopy can be seen in our recent measurements of the low field ($\nu > 2$) emission. An example of a PL spectra from sample A is shown in Fig. 7. At filling factor $\nu = 2.7$ two well-defined features can be seen in the spectra below the fundamental band of recombination from the lowest Landau level. We should mention that these observed peaks have no analog in absorption, indicating the importance of relaxation and many-body effects in their origin. Similar behavior has very recently been observed by Finkelstein et. al. [16]. Finkelsten et al. have constructed a theory that appears to account well for these observations. We follow their convention and label the low-energy peaks SU_0 and SU_1 as the 0th and 1st shake-up peaks below the fundamental recombination line, respectively. These peaks are attributed to a shake-up process in a magnetic field where a recombining electron–hole pair perturbs the surrounding 2DES resulting in a reduced emitted photon energy. In this scheme the SU_0 process results from the recombination of an electron from LL_1 with the valence band hole accompanied by an excitation of another electron across the cyclotron gap. Finally, in Fig. 8 we display the field dependence of the shake-up lines. All lines

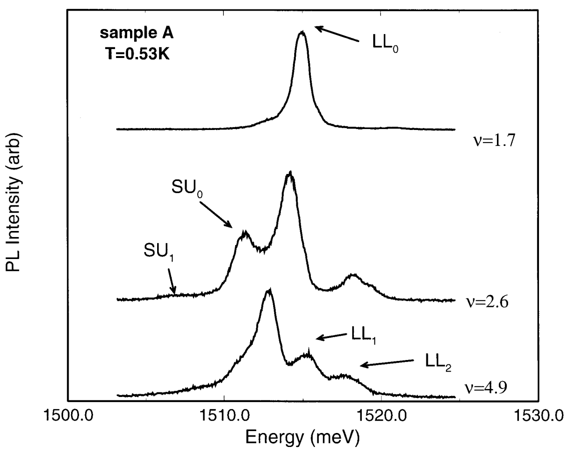

Fig. 8. Filling-factor dependence of the shakeup lines in sample A at $T = 0.53$ K. At fields corresponding to $\nu < 2$ the shakeup lines are greatly suppressed.

evolve very rapidly in field and are largely suppressed at filling factors $\nu < 2$, further indicating the complicated interplay of level population, and changing matrix elements and relaxation effects for the emission process.

4. Summary

In conclusion, we have shown the ability of magneto-absorption spectroscopy to accurately probe the occupancy of the two spin bands of the lowest Landau level. The spin polarization of the 2DES shows a rapid, and quasi-symmetric, quenching for deviations from $\nu = 1$. Data presented from three wafers are consistent with a Skyrmion model for the quasi-particles of the ferromagnetic ground state at $\nu = 1$. Additionally, the temperature dependence of the spin polarization at $\nu = 1$ is consistent with the thermodynamic model of Read and Sachdev and shows the importance of spin-wave–spin-wave interactions. Finally, we have reported some very recent results of emission studies in the quantum Hall regime and discuss some of the differences that exist between absorption and emission spectroscopy.

Acknowledgements

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