

Hidden Quantum Hall Stripes in  $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$  Quantum WellsX. Fu,<sup>1</sup> Yi Huang (黄奕)<sup>1</sup>, Q. Shi,<sup>1,†</sup> B. I. Shklovskii,<sup>1</sup> M. A. Zudov,<sup>1,\*</sup> G. C. Gardner,<sup>2,3</sup> and M. J. Manfra<sup>2,3,4,5</sup><sup>1</sup>*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*<sup>2</sup>*Microsoft Quantum Lab Purdue, Purdue University, West Lafayette, Indiana 47907, USA*<sup>3</sup>*Birck Nanotechnology Center, Purdue University, West Lafayette, Indiana 47907, USA*<sup>4</sup>*Department of Physics and Astronomy, Purdue University, West Lafayette, Indiana 47907, USA*<sup>5</sup>*School of Electrical and Computer Engineering and School of Materials Engineering, Purdue University, West Lafayette, Indiana 47907, USA*

(Received 23 August 2020; accepted 30 October 2020; published 2 December 2020)

We report on transport signatures of hidden quantum Hall stripe (hQHS) phases in high ( $N > 2$ ) half-filled Landau levels of  $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$  quantum wells with varying Al mole fraction  $x < 10^{-3}$ . Residing between the conventional stripe phases (lower  $N$ ) and the isotropic liquid phases (higher  $N$ ), where resistivity decreases as  $1/N$ , these hQHS phases exhibit isotropic and  $N$ -independent resistivity. Using the experimental phase diagram, we establish that the stripe phases are more robust than theoretically predicted, calling for improved theoretical treatment. We also show that, unlike conventional stripe phases, the hQHS phases do not occur in ultrahigh mobility GaAs quantum wells but are likely to be found in other systems.

DOI: 10.1103/PhysRevLett.125.236803

Discovery of the integer quantum Hall effect in Si [1] has paved the way to observations of many exotic phenomena in two-dimensional (2D) electron and hole systems. Two prime examples are the fractional quantum Hall effect [2] and quantum Hall stripes (QHSs) [3–7]. While fractional quantum Hall effects have been realized in many systems, including GaAs [2], Si [8,9], AlAs [10], GaN [11], graphene [12,13], CdTe [14], ZnO [15], Ge [16], and InAs [17], exploration of the QHS physics remains limited to GaAs [18].

Forming due to a peculiar boxlike screened Coulomb potential, QHSs can be viewed as charge density waves consisting of stripes with alternating integer filling factors  $\nu$ , e.g.,  $\nu = 4$  and  $\nu = 5$  [25]. In experiments, QHSs are manifested by giant resistivity anisotropies ( $\rho_{xx} \gg \rho_{yy}$ ) in  $N \geq 2$  half-filled Landau levels. Appearance of these anisotropies in macroscopic samples is attributed to a mysterious symmetry-breaking field [27–30], which nearly always aligns QHSs along the  $\hat{y} \equiv \langle 110 \rangle$  crystal axis of GaAs [31]. While a sufficiently low disorder is necessary for the QHS formation, the absence of QHSs in systems beyond GaAs might simply be due to the lack of symmetry-breaking fields [33]. Indeed, electron bubble phases [3–5, 43–52], which are close relatives of QHSs, have already been identified in graphene [53].

In this Letter, we report observations of transport signatures of the recently predicted [54] hidden QHS (hQHS) phases in a series of  $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$  quantum wells with  $x < 10^{-3}$ . In contrast to the ordinary QHS phases, the hQHS phases are characterized by isotropic resistivity ( $\rho_{xx} = \rho_{yy} = \rho$ ) that is independent

of  $\nu$ , unlike the isotropic liquid phases in which  $\rho \propto \nu^{-1}$ . These unique properties make these phases detectable without symmetry-breaking fields, thereby opening an avenue to study stripe physics in systems beyond GaAs. The wide variation of mobilities in our samples allows us to construct an experimental phase diagram in the conductivity-filling factor plane. Its comparison to theoretical predictions [54] yields the electron quantum lifetimes and the stripe density of states. The latter turns out to be lower than predicted by the original Hartree–Fock theory [3,4], calling for further theoretical input. We confirm this finding by a complementary experiment on an ultrahigh mobility GaAs quantum well, where we also show that, in this sample, the hQHS phase yields to the QHS phase in agreement with the theory.

Before presenting our experimental data, we briefly summarize the physics behind the hQHS phases [54]. The resistance anisotropies in the ordinary QHS phase emerge due to different diffusion mechanisms along and perpendicular to the stripes [55–57]. In this picture, an electron drifts a distance  $L_y$  along the  $y$ -oriented stripe edge in an  $x$ -directed internal electric field until it is scattered by impurities to one of the adjacent stripe edges located at a distance  $L_x = \Lambda/2 \approx \sqrt{2}R_c$  [3,4], where  $\Lambda$  is the stripe period and  $R_c$  is the cyclotron radius. When  $L_y \gg L_x$ , the diffusion coefficient in the  $\hat{y}$  direction is much larger than in the  $\hat{x}$  direction, which leads to anisotropic conductivity,  $\sigma_{yy} \gg \sigma_{xx}$ , and resistivity,  $\rho_{xx} \gg \rho_{yy}$ . Since  $L_y \propto \nu^{-1}$  and  $L_x \propto \nu$  [57], the anisotropy decreases with  $\nu$  and eventually vanishes at some  $\nu = \nu_1$ . At larger  $\nu$ , the drift contribution

to the diffusion along stripes can be neglected, and  $L_y$ , like  $L_x$ , is determined entirely by the impurity scattering. For isotropic scattering, it is easy to show [58] that  $L_y = \sqrt{2}R_c$  which coincides with  $L_x$ . As a result, the QHS phase yields to the hQHS phase in which the resistivity is isotropic and  $\nu$  independent (since the stripe density of states does not vary with  $\nu$ ). The hQHS phase persists until the stripe structure is destroyed by disorder at  $\nu = \nu_2$  and the ground state becomes an isotropic liquid with  $\rho_{xx} = \rho_{yy} \propto \nu^{-1}$ , as predicted by Ando and Uemura [59] and experimentally confirmed by Coleridge, Zawadski, and Sachrajda (CZS) [60].

For the hQHS phase to exist and be detected, it should span a sizable range of the filling factors  $\Delta\nu = \nu_2 - \max\{\nu_1, 9/2\} \gg 1$ . The range  $\Delta\nu$  depends sensitively on both transport  $\tau^{-1}$  and quantum  $\tau_q^{-1}$  scattering rates, which control  $\nu_1$  and  $\nu_2$ , respectively [54]. As we will see, ultrahigh mobility GaAs quantum wells do not support the hQHS phase as  $\nu_1 \approx \nu_2$  in these samples. On the other hand, adding the correct small amount of Al [61] to the GaAs well greatly expands  $\Delta\nu$ , as it affects  $\nu_1$  to a much greater extent than it does  $\nu_2$ . This happens because Al acts as a short-range disorder, which contributes *equally* to transport  $\tau^{-1}$  and quantum  $\tau_q^{-1}$  scattering rates, and because  $\tau_q/\tau \ll 1$  at  $x = 0$ .

Apart from different  $x$ , all our  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  quantum wells share an identical heterostructure design [62]. Electrons are supplied by Si doping in narrow GaAs wells surrounded by narrow AlAs layers and placed at a setback distance of 75 nm from each side of the 30-nm-wide  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  well hosting the 2D electrons. Parameters of samples A, B, and C such as Al mole fraction  $x$ , electron density  $n_e$ , mobility  $\mu$ , and Drude conductivity  $\tilde{\sigma}_0 = \hbar n_e \mu / e$  in units of  $e^2/h$  at zero magnetic field ( $B = 0$ ) are listed in Table I. The samples are approximately 4 mm squares with eight indium contacts positioned at the corners and at the midsides. Longitudinal resistances  $R_{xx}$  and  $R_{yy}$  were measured in sweeping magnetic fields using a four-terminal, low-frequency (a few Hz) lock-in technique at a temperature  $T \approx 25$  mK at which the resistances are nearly temperature independent. The current was sent along either the  $\hat{x} \equiv \langle 1\bar{1}0 \rangle$  or  $\hat{y} \equiv \langle 110 \rangle$  direction using the midside contacts, and the voltage was measured between contacts along the edge. To account for anisotropies due to nonideal

TABLE I. Sample ID, Al mole fraction  $x$ , electron density  $n_e$ , mobility  $\mu$ , and Drude conductivity in units of  $e^2/h$ ,  $\tilde{\sigma}_0 = \hbar n_e \mu / e$  at zero magnetic field ( $B = 0$ ).

Sample ID	$x$	$n_e$ ( $10^{11} \text{ cm}^{-2}$ )	$\mu$ ( $10^6 \text{ cm}^2 \text{ Vs}$ )	$\tilde{\sigma}_0$ ( $10^3$ )
A	0.000 57	3.0	6.5	8.0
B	0.000 82	2.9	4.1	4.9
C	0.0078	2.7	1.2	1.3

geometry,  $R_{xx}$  or  $R_{yy}$  was multiplied by a factor (typically  $\lesssim 1.1$ ) that was chosen to make  $R_{xx} = R_{yy}$  in the low field regime.

In Fig. 1, we present longitudinal resistances  $R_{xx}$  and  $R_{yy}$  as a function of filling factor  $\nu$  measured in sample B. At low half-integer filling factors,  $\nu = 9/2, 11/2$ , and  $13/2$ , the data reveal conventional QHS phases, as evidenced by  $R_{xx} > R_{yy}$ . At high half-integer filling factors,  $\nu > 25/2$ , we identify the CZS phase in which  $R_{xx} \approx R_{yy} \propto \nu^{-1}$  (cf. dash-dotted line). At intermediate half-integer filling factors,  $\nu = 15/2, \dots, 23/2$ , one readily confirms *both* characteristic features of the hQHS phase; indeed, the data show that two longitudinal resistances are practically the same ( $R_{xx} \approx R_{yy}$ ) and are *independent* of  $\nu$  (cf. dashed line). From Fig. 1, we can easily identify the characteristic filling factors  $\nu_1 \approx 7$  and  $\nu_2 \approx 12.5$ , which mark the crossovers from the QHS to the hQHS phase and from the hQHS to the CZS phase, respectively.

In a similar manner, we have obtained  $\nu_1$  and  $\nu_2$  for sample A and  $\nu_2$  for sample C (which does not support the QHS phase due to higher Al mole fraction  $x$ ), which we then use to construct the experimental phase diagram shown in Fig. 2. We start by adding points representing the dimensionless Drude conductivity  $\tilde{\sigma}_0$  for samples A, B, C (see Table I) and the corresponding filling factors  $\nu_1$  (solid

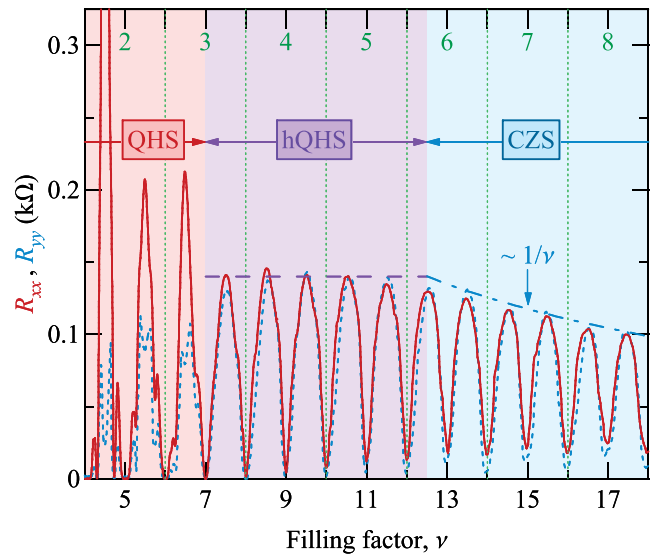


FIG. 1. Longitudinal resistances  $R_{xx}$  (solid line) and  $R_{yy}$  (dotted line) as a function of the filling factor  $\nu$  measured in sample B. Gap centers between spin-resolved Landau levels are labeled by  $N = 2, 3, \dots$ , at the top axis ( $\nu = 2N + 1$ ). The conventional QHS phase ( $R_{xx} > R_{yy}$ ) and the CZS phase ( $R_{xx} \approx R_{yy} \propto \nu^{-1}$ ) occur at half-integer  $\nu = 9/2, 11/2, 13/2$  and at  $\nu = 27/2, 29/2, \dots$ , respectively. The hQHS phase is identified at intermediate half-integer filling factors  $\nu = 15/2, \dots, 25/2$ , where the resistance is isotropic *and*  $\nu$  independent. The characteristic  $\nu^0$  ( $\nu^{-1}$ ) dependence of the isotropic resistance in the hQHS (CZS) phase is marked by dashed (dash-dotted) line.

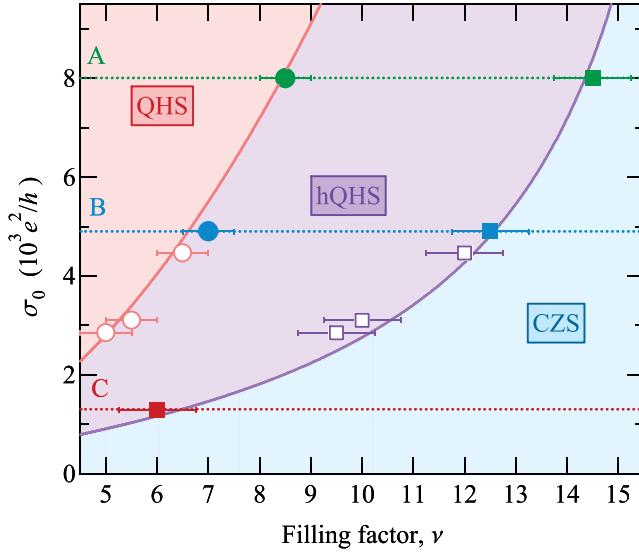


FIG. 2. A diagram in the  $(\nu, \sigma_0)$  plane showing QHS, hQHS, and CZS phases. Solid lines represent crossovers between phases, Eq. (1) [left (upper) line] and Eq. (3) [right (lower) line]. Solid circles (solid squares) represent experimental  $\nu_1$  ( $\nu_2$ ) and horizontal dotted lines mark  $\tilde{\sigma}_0$  for samples A–C [62]. Open circles (squares) are additional data from a study conducted in a different context that conform to our present findings [64].

circles) and  $\nu_2$  (solid squares). To connect these data points, we use the theoretical boundaries of the hQHS phase [54]. The lower boundary  $\nu = \nu_1$ , separating the QHS and the hQHS phases, is given by [54]

$$\nu_1 \simeq \frac{\sqrt{\tilde{\sigma}_0}}{\alpha}, \quad (1)$$

where  $\alpha$  [63] is the QHS density of states in units of the density of states per spin at  $B = 0$ ,  $g_0 = m^*/2\pi\hbar^2$ . This boundary can be obtained by either matching the parameter-free geometric average of the resistivities in the QHS phase  $\sqrt{\rho_{xx}\rho_{yy}} = (h/e^2)/(2\nu^2 + 1/2) \approx (h/e^2)/2\nu^2$  [55,57] and the resistivity in the hQHS phase [54],

$$\tilde{\rho}_{\text{hQHS}} \equiv \frac{h}{e^2} \frac{\alpha^2}{2\tilde{\sigma}_0}, \quad (2)$$

or, equivalently, by setting the resistivity anisotropy ratio to unity,  $\rho_{xx}/\rho_{yy} \approx (\tilde{\sigma}_0/\alpha^2\nu^2)^2 = 1$  [54,57].

The higher boundary  $\nu = \nu_2$  marks the crossover from the hQHS to the CZS phase and is represented by

$$\nu_2 \simeq \frac{\tilde{\sigma}_0 \tau_q}{\alpha^2 \tau}. \quad (3)$$

This boundary can be obtained by equating  $\alpha$  and the density of states at the center of the Landau level in CZS phase in units of the density of states at  $B = 0$ ,

$\sqrt{\omega_c \tau_q}$  [65,66] or by matching  $\rho_{\text{hQHS}}$  and the resistivity in the CZS phase [60],

$$\rho_{\text{CZS}} \equiv \frac{h}{e^2} \frac{1}{\nu} \frac{\tau_q/2\tau}{(\tau_q/2\tau)^2 + 1} \approx \frac{h}{e^2} \frac{1}{\nu} \frac{\tau_q}{2\tau}. \quad (4)$$

We thus see that for a given carrier density, as mentioned above,  $\nu_2$  and  $\nu_1$  are controlled by  $\tau$  and  $\tau_q$ , respectively. Strictly speaking, Eqs. (1) and (3) are not sharp boundaries but rather gradual crossovers between corresponding phases.

With the help of Eq. (1) and experimental values of  $\nu_1$  in samples A and B, we estimate  $\alpha \approx 11$ , which is smaller than the theoretical estimate of  $\alpha \approx 18$  [57,63]. We then parameterize scattering rates  $\tau^{-1}$  and  $\tau_q^{-1}$  as

$$\tau^{-1} = \tau_0^{-1} + \kappa x, \quad \tau_q^{-1} = \tau_{q0}^{-1} + \kappa x, \quad (5)$$

where  $x$  is the Al mole fraction,  $\kappa \approx 24 \text{ ns}^{-1}$  per % Al [62], and  $\tau_0^{-1} \approx 3 \text{ ns}^{-1}$  [62] is the transport scattering rate in the limit of  $x \rightarrow 0$ . To find the remaining parameter  $\tau_{q0}^{-1}$ , which is the quantum scattering rate in the limit of  $x \rightarrow 0$ , we use experimental  $\nu_2$  values and notice that Eqs. (1) and (3) yield  $\tau_q/\tau \approx \nu_2/\nu_1^2$ . Using Eq. (5), we then obtain an estimate for  $\tau_{q0} \approx 0.05 \text{ ns}$ , which is in good agreement with  $\tau_q$  values found from low  $B$  experiments [67–69] on microwave-induced [70–72] and Hall-field-induced [73–75] resistance oscillations in GaAs quantum wells.

We next use  $n_e = 3 \times 10^{11} \text{ cm}^{-2}$  and  $m^* = 0.06m_0$  [76–80] to compute the phase boundaries, Eqs. (1) and (3), which are shown in Fig. 2 by solid lines. Both lines pass in close proximity to the experimentally obtained  $\nu_1$  (solid circles) and  $\nu_2$  (solid squares) from all samples, showing excellent agreement between theory [54] and experiment. Finally, we add data points (open circles and squares) from three other  $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$  quantum wells that were investigated in a different context [64]. These points are also in agreement with the theory and the present experiment.

Having confirmed the existence of the hQHS phases in  $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$  quantum wells, we next examine the possibility for these phases to exist in ultrahigh mobility GaAs quantum wells (without alloy disorder). In such samples, the lower boundary  $\nu_1$ , Eq. (1), might approach and even merge with the higher boundary  $\nu_2$ , Eq. (3), eliminating the hQHS phase as a result. To test this scenario, we revisit the data obtained from sample A of Ref. [57] with  $\tilde{\sigma}_0 \approx 3.4 \times 10^4$ , much higher than in samples used in the present study. As illustrated in Fig. 3, showing  $\rho_{xx}$  (solid triangles) and  $\rho_{yy}$  (open triangles) [81] as a function of the filling factor  $\nu$ , the QHS anisotropy in this sample collapses at  $\nu_1 \approx 20$ . Using Eq. (1), we can then estimate  $\alpha = \sqrt{\tilde{\sigma}_0}/\nu_1 \approx 9$  [83]. With  $\tau_q \approx 0.05 \text{ ns}$ , Eq. (3) gives  $\nu_2 \approx 21$ , which is very close to  $\nu_1 \approx 20$ . Indeed, the

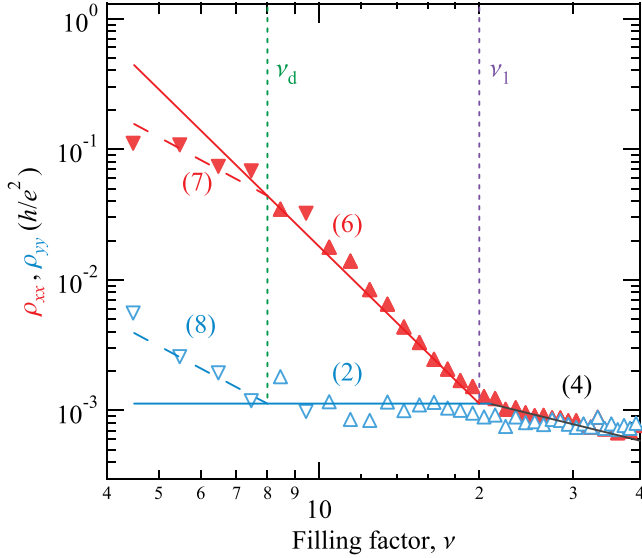


FIG. 3.  $\rho_{xx}$  (solid triangles) and  $\rho_{yy}$  (open triangles) [81] as a function of filling factor  $\nu$  for sample A of Ref. [57]. Lines are computed using theoretical expressions, marked by equation numbers.

data in Fig. 3 show that the QHS phase crosses over directly to the CZS phase, bypassing the intermediate hQHS phase.

In the QHS phase, the easy resistivity is  $\nu$  independent and is described by  $\rho_{yy} = \rho_{\text{hQHS}}$ , Eq. (2), while the hard resistivity exhibits clear  $\nu^{-4}$  dependence and follows [54]

$$\rho_{xx} \simeq \frac{h}{e^2} \frac{\tilde{\sigma}_0}{2\alpha^2 \nu^4}. \quad (6)$$

However, the agreement between theory and experiment breaks down at  $\nu < \nu_d \approx 8$ , where one observes significant deviations leading to the reduction of the anisotropy. While the nature of such reduction is unclear, it becomes more pronounced upon further cooling and might reflect a crossover to another competing ground state [85,86]. We can account for the observed anisotropy reduction at lower filling factors assuming that the QHS phase has a finite concentration of dislocations separated by an average distance  $L_d = \beta\Lambda/2$  along stripes, where  $\beta$  is a numerical factor. Scattering of drifting electrons by these dislocations limits their drift length by  $L_d \ll L_y$  and the resistivities calculated in [54,57] need to be modified to [87,88]

$$\rho_{xx} = \frac{h}{e^2} \frac{\beta}{2\nu^2}, \quad (7)$$

$$\rho_{yy} = \frac{h}{e^2} \frac{1}{2\beta\nu^2}. \quad (8)$$

Equations (7) and (8) are plotted as dashed lines in Fig. 3. Equating Eq. (7) to Eq. (6) [or Eq. (8) to Eq. (2)], we find that the crossover to the dislocation limited transport happens at

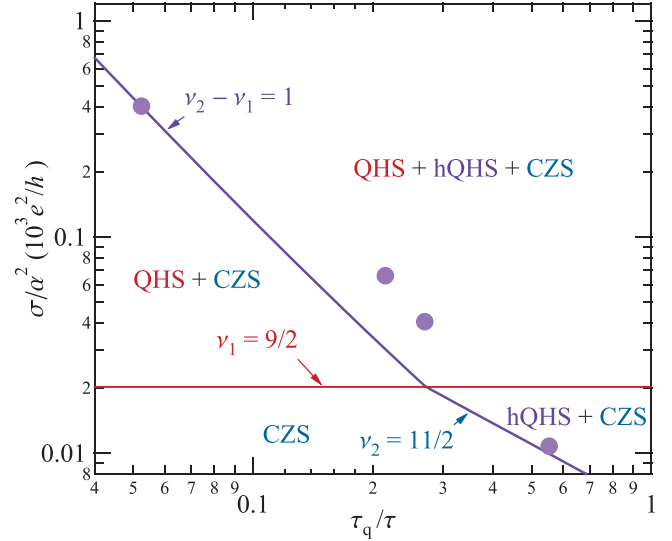


FIG. 4. A diagram in the  $(\tau_q/\tau, \sigma_0/\alpha^2)$  plane showing four regions marked by detectable phases. Circles are experimental data points from all four samples studied.

$$\nu_d \equiv \frac{\nu_1}{\sqrt{\beta}}. \quad (9)$$

With  $\nu_d \simeq 8$  and  $\nu_1 \simeq 20$ , we find  $\beta = (\nu_1/\nu_d)^2 \simeq 6.3$ . This value does not seem unreasonable and correctly accounts for the saturation of the anisotropy,  $\rho_{xx}/\rho_{yy} = \beta^2 \approx 40$  [57].

Our experimental findings in  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  quantum wells (Fig. 2) and in a clean GaAs quantum well (Fig. 3) can be unified in a phase diagram shown in Fig. 4, which treats  $\sigma_0/\alpha^2$  and  $\tau_q/\tau$  as independent parameters. Here, the QHS phase is observed above the horizontal line corresponding to  $\nu_1 = 9/2$ . To detect the hQHS phase, one should satisfy both  $\nu_2 - \nu_1 > 1$  and  $\nu_2 > 11/2$ , since at least two half-integer filling factors are needed to establish the  $\nu$  independence of the resistance [89]. As a result, the most favorable conditions for the hQHS phase are realized at the top-right corner of the diagram. However, as demonstrated by our experiments on  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  quantum wells, the hQHS can be detected at modest mobilities provided that the ratio  $\tau_q/\tau$  is sufficiently high. On the other hand, this ratio is much smaller in clean GaAs quantum wells, which makes the hQHS detection difficult in such systems despite their high mobility. The phase diagram shown in Fig. 4 provides a road map for future experiments aiming to detect the hQHS phases.

In summary, we have observed hidden quantum Hall stripe (hQHS) phases [54] forming near half-integer filling factors of  $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$  quantum wells with varying  $x$ . These phases reside between the conventional stripe phases and the isotropic liquid phases and are characterized by isotropic resistivity that is not sensitive to the filling factor. Analysis of the experimental phase diagram reveals that the QHS density of states is smaller



than predicted by the Hartree–Fock theory [3,4], calling for improved theory. The unique transport characteristics of the hQHS phases should allow exploration of the stripe physics in 2D systems that, unlike GaAs, lack symmetry-breaking fields. On the other hand, ultrahigh mobility GaAs quantum wells favor conventional QHSs over hQHSs due to a shrinking filling factor range where the hQHS phases can form.

We thank G. Jones, T. Murphy, and A. Bangura for technical support. Calculations by Y. H. were supported primarily by the National Science Foundation through the University of Minnesota MRSEC under Grant Nos. DMR-1420013 and DMR-2011401. Experiments by X. F., Q. S., and M. Z. were supported by the U.S. Department of Energy, Office of Science, Basic Energy Sciences, under Award DE-SC0002567. Growth of quantum wells at Purdue University was supported by the U.S. Department of Energy, Office of Science, Basic Energy Sciences, under Award DE-SC0006671. X. F. acknowledges the University of Minnesota Doctoral Dissertation Fellowship. A portion of this work was performed at the National High Magnetic Field Laboratory, which is supported by National Science Foundation Cooperative Agreement Nos. DMR-1157490 and DMR-1644779, and the State of Florida.

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- With dislocations,  $L_y \propto L_x \propto \nu$ , i.e.,  $\tilde{\sigma}_{jj} \propto \nu^0$ , and  $\tilde{\rho}_{xx} \propto \tilde{\rho}_{yy} \propto \nu^{-2}$ .
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- [89] The approximate condition for the hQHS phase detection is  $\tilde{\sigma}_0 > \alpha^2(\tau/\tau_q) \max\{\tau/\tau_q, 11/2\}$ .