Phase engineering of anomalous Josephson effect derived from Andreev molecules

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A Josephson junction (JJ) is a key device for developing superconducting circuits, wherein a supercurrent in the JJ is controlled by the phase difference between the two superconducting electrodes. When two JJs sharing one superconducting electrode are coherently coupled and form the Andreev molecules, a supercurrent of one JJ is expected to be nonlocally controlled by the phase difference of another JJ. Here, we evaluate the supercurrent in one of the coupled two JJs as a function of local and nonlocal phase differences. Consequently, the results exhibit that the nonlocal phase control generates a finite supercurrent even when the local phase difference is zero. In addition, an offset of the local phase difference giving the JJ ground state depends on the nonlocal phase difference. These features demonstrate the anomalous Josephson effect realized by the nonlocal phase control. Our results provide a useful concept for engineering superconducting devices such as phase batteries and dissipationless rectifiers.

INTRODUCTION

Symmetry-breaking superconducting (SC) junctions have often been used to explore exotic SC phenomena including SC diodes (1–3), topological superconductivity, and Majorana zero modes (4). The Josephson junction (JJ) (5) is an SC device that is often used for searching such SC phenomena. The JJs exhibit anomalous Josephson effect (AJE) upon the disorientation of the time-reversal and spatial-inversion symmetries, in which a finite phase difference away from 0 and π produces the ground state of the JJ, called the Φ junction (6–14). Simultaneously, a finite supercurrent is present even at zero phase difference, called the spontaneous supercurrent. The Φ junction has recently attracted considerable attention for manifold applications in SC phase batteries (11, 15) and SC diodes (1, 16–19). To date, various systems have been proposed for realizing the Φ junctions, such as JJs consisting of s-wave SC electrodes and a normal metal holding the spin-orbit interactions in strong magnetic fields (6–10), and have been experimentally verified as well (11–14). When the strong magnetic fields or ferromagnetism is committed to disintegrate the time-reversal symmetry, they can degrade the superconductivity. On the other hand, the phase control of the SC devices also breaks the time-reversal symmetry and is available to realize the SC diode effect (20–23). This implies that the phase control is useful to engineer the AJE in the SC devices.

Recently, it has been proposed that short-range coherent coupling of two JJs sharing one SC electrode can hybridize the Andreev bound states (ABSs) (24–27) in the respective JJs to form Andreev molecule states (AMSs) (28–34). The coherent coupling through the shared SC electrode is intermediated by elastic cotunneling or crossed Andreev reflection (35–43), and the SC transport has been studied theoretically and experimentally (28, 44–49). The AMSs in the coupled two JJs generate the nonlocal Josephson effect, wherein the supercurrent in a JJ depends on the local phase difference as well as the nonlocal phase difference of the other JJ (28).

In the coupled JJs illustrated in Fig. 1A, the supercurrent in the JJ1 satisfies the time-reversal relation of $I_{\text{sc},1}(\phi_1, \phi_2) = -I_{\text{sc},1}(\phi_1, -\phi_2)$, where $\phi_1$ and $\phi_2$ denote the phase differences in JJ1 and JJ2, respectively. When the SC phases of the upper, shared, and lower electrodes are defined as $\theta_u$, $\theta_s$, and $\theta_l$, respectively, the local and nonlocal phase differences of $\phi_1$ and $\phi_2$ are given as $\phi_1 = \theta_u - \theta_s$ and $\phi_2 = \theta_l - \theta_s$. Upon setting $\phi_2$ and considering only JJ1, a character of the $\Phi$ junction can emerge (28) because the time-reversal and spatial inversion symmetries can be regarded as disintegrated at $\phi_2 \neq 0, \pi$. This realization method requires neither strong magnetic fields nor ferromagnetic materials. Furthermore, the $\Phi$ junction and spontaneous supercurrent of JJ1 obtained by the mechanism are nonlocally controllable through JJ2, which will provide a useful method to control phase batteries or SC diodes. We have previously demonstrated the SC diode effect derived from the coherent coupling, namely, the AMSs (45). However, the AJE (spontaneous supercurrent and $\Phi$ junction) has not yet been experimentally verified.

In this study, we succeed in evaluating the supercurrent in JJ1 as a function of $\phi_1$ and $\phi_2$. The obtained results exhibit the supercurrent dependent on not only $\phi_1$ but also $\phi_2$. In the dependence, the spontaneous supercurrent and $\Phi$ junction derived from the AMSs formed in the coupled two planar JJs are found. It is predicted that although the topological superconductivity is engineered in single planar JJs with the phase control and the Zeeman field (50–52), the coherent coupling of two planar JJs engineers the topological superconductivity only with the phase control (53, 54). For the realization, it is demanded to elucidate the fundamental physics of the AMSs in the coupled JJs, especially about the symmetry
breaking invoked by the phase control. In this sense, the phase-engineering of the AJE in the planar JJs contributes to realizing the topological superconductivity only by the phase control.

To demonstrate the phase engineering of the AJE, we need to evaluate the two-dimensional current phase relation (CPR) \( I_{eg}(\phi_1, \phi_2) \), i.e., the supercurrent in JJ1, as a function of the local and nonlocal phase differences. For this sake, we use asymmetric SC quantum interference devices (SQUIDs) (55) whose scanning electron microscopy (SEM) image and schematic are shown in Fig. 1 (B and C), respectively. The device is fabricated on a high-quality InAs quantum well covered with an epitaxial aluminum thin film. The stacking of the epitaxial aluminum and the InAs quantum well provides a highly transparent interface to provide an ideal platform for studying the physics of superconductor-semiconductor junctions (56–58). The device includes the coupled JJs (JJ1 and JJ2) with a shared SC electrode 150 nm in width. The separation between JJ1 and JJ2 is sufficiently shorter than the coherence length of aluminum (~1 \( \mu \)m). We note that the AMSs have been demonstrated in coupled JJs with the same separation (32). The junction length and width of the JJ1 and JJ2 are 100 and 600 nm, respectively. Furthermore, two larger JJs, named JJL1 and JJL2 with 2-\( \mu \)m width and 100-nm length, are prepared to form larger and smaller asymmetric SQUIDs. Subsequently, we place the gate electrodes on all the JJs to control them by the gate voltages. As depicted in Fig. 1C, we bias an electrical current \( I_2 \) when measuring the larger SQUID and measure the voltage difference \( V_1 \) as well. All the measurements were performed at 10 mK of the base temperature in our dilution refrigerator.

**RESULTS**

First, we characterize the single JJ properties of JJ1 and JJ2 with the other JJs pinched off (see note S1 and fig. S1). The current-voltage relation (I-V) curve of JJ1 at an out-of-plane magnetic field \( B = 0 \) mT with \( V_{g1} = -1.4 \) V of the gate voltage for JJ1 with the other JJs pinched off is portrayed in Fig. 1D, wherein the supercurrent flows and the switching current of JJ1 is 0.2 \( \mu \)A. The switching...
current of JJ1 is highly tunable as shown in Fig. 1E. The JJ2 indicates a similar switching current and the dependence on the gate voltage $V_{g2}$ for JJ2 as shown in Fig. 1F.

As the first step, we evaluate the CPR of the single JJ1 by measuring $V_1$ and $I_1$ of the larger asymmetric SQUID with $V_{g1} = -1.4$ V, JJ2 off, $V_{g1} = -1.4$ V, and JJL2 off. For the evaluation, $V_1$ as a function of $I_1$ and $B$ are measured to obtain the switching current of the asymmetric SQUID. The asymmetric SQUID is used to evaluate the CPR because the switching current is approximately written as $I_{sc1} [\phi_{1c} + 2 \pi \Phi(B)/\Phi_0] + I_{swL1} (27, 55)$. Here, the single JJ1 CPR and the switching current of the single JJ1 are $I_{sc1} [\phi_{1c} + 2 \pi \Phi(B)/\Phi_0]$ and $I_{swL1}$, respectively. $\phi_{1c}$ and $\Phi_0 = \hbar/2e$ the constant phase difference decided by the critical current of JJ1 and the loop inductance and flux quantum, respectively. $\Phi(B)$ represents the magnetic flux in the larger loop, which depends on $B$ and the loop area. Therefore, we subtract the background assigned to the Fraunhofer-type interference in JJL1 to obtain the single JJ1 CPR (see note S2 and fig. S2). Consequently, the single JJ1 CPR curve, $I_{sc1} [\phi_{1c} + 2 \pi \Phi(B)/\Phi_0]$, is obtained as shown with black circles in Fig. 1G. We assume that the correction from the inductance and circulating supercurrent in the loop is ignorable, and the magnetic flux in the loops is linearly dependent on $B$ (see note S4 and fig. S4). The CPR periodically oscillates, and its shape is skewed from the sinusoidal function of $B$, reflecting the short ballistic nature of JJ1. These results assure that our asymmetric SQUID can be used to evaluate the JJ1 CPR.

Then, the CPR of JJ1 coupled with JJ2 is studied with $(V_{g1}, V_{g2}) = (-1.4$ V, $-1.45$ V) and $(V_{g1}, V_{g2}) = (-1.4$ V, $-1.15$ V). These gate voltages produce similar switching currents of 0.2 $\mu$A in JJ1 and JJ2. For this sake, $V_1$ obtained as a function of $I_1$ and $B$ is measured with JJL1 and JJL2 on. The coupled JJ1 CPR curve obtained by subtracting the same background data as the single JJ1 CPR evaluation is shown as green circles in Fig. 1G. The obtained CPR curve is highly modulated from that of JJ1 with no JJ2 and is not periodic. This modulation originates from the AMS formation in the coupled JJs, which makes the CPR of JJ1 dependent on $\phi_2$. Now, $B$ changes not only $\phi_1$ but also $\phi_2$ because both of the asymmetric SQUIDs are formed. Then, $\phi_2$ is introduced as $2 \pi \Phi_2 (B)/\Phi_0$, where $\Phi_2 (B)$ represents the magnetic flux in the smaller loop. As $[\phi_1] < [\phi_2]$ holds from the loop area relation, $\phi_1$ and $\phi_2$ evolve with $B$ at the different ratios. Therefore, the $[\phi_1(B), \phi_2(B)]$ function is depicted in Fig. 1H.

For instance, if $B$ increases from $\phi_1 = \phi_2 = 0$, the trace of $(\phi_1, \phi_2) = [\phi_{1c} + 2 \pi \Phi_1 (B)/\Phi_0, 2 \pi \Phi_2 (B)/\Phi_0]$ moves with $B$ from $(\phi_{1c}, 0)$ along the solid arrows in Fig. 1H. Reaching at $\phi_1 = \pi$, the trace is shifted to $\phi_1 = -\pi$ with the same $\phi_2$ and moves along the solid arrow. The described solid lines correspond to the trace when $\phi_1$ changes by $7 \times 2\pi$. We note that $-0.9 \text{ mT} < B < 0.3 \text{ mT}$ includes around seven periods in $\phi_1$ seen in the single JJ1 CPR curve in Fig. 1G. Therefore, the green circles in Fig. 1G represent $I_{sc1} [\phi_{1c} + 2 \pi \Phi_1 (B)/\Phi_0, 2 \pi \Phi_2 (B)/\Phi_0]$, and the nonperiodic dependence on $B$ indicates that the JJ1 CPR depends on $\phi_2$ due to the AMS formation.

The trace lines do not fill the $(\phi_1, \phi_2)$ plane in Fig. 1H. Therefore, only a portion of the two-dimensional CPR can be constructed from the green circles in Fig. 1G. A way to fill the entire $(\phi_1, \phi_2)$ plane is to shift the trace lines along the vertical $\phi_2$ axis and obtain the $[\phi_{1c} + 2 \pi \Phi_1 (B)/\Phi_0, 2 \pi \Phi_2 (B)/\Phi_0 + \Delta \phi_2]$ traces. Here, $\Delta \phi_2$ represents the shift of $\phi_2$. For this sake, we add the bias current $I_2$ in the smaller SQUID because the finite supercurrent in the asymmetric SQUID shifts the phase differences of JJs following their CPRs and the SC loop inductance. To evaluate $\Delta \phi_2$ induced by $I_2$, we measure the JJ1 CPR as a function of $I_2$ and $B$ at $-0.6 \mu A \leq I_2 \leq 0.6 \mu A$ when JJ2 and JJL2 are on. The oscillation of switching current originates from the nonlocal Josephson effect derived from the coupling of JJ1 and JJ2. The curves shift along the $B$ axis as $I_2$ varies, implying that the phase shift of JJ2 ($\Delta \phi_2$) is induced. (B) Evaluated $\Delta \phi_2$ as a function of $I_2$ from (A). $\Delta \phi_2$ is tunable with $I_2$ in the range of $-0.2 \pi < \Delta \phi_2 < 0.2 \pi$. (C) The obtained JJ1 supercurrent as a function of $B$ at $-0.6 \mu A \leq I_2 \leq 0.6 \mu A$. Line colors specifying $I_2$ are consistent with those in (A). The curve shapes are modulated by changing $I_2$.  

Fig. 2. Necessary dataset to obtain the two-dimensional CPR of JJ1 coupled to JJ2. (A) Switching current of JJ1 ($I_{swL1}$) as a function of $B$ at $-0.6 \mu A \leq I_2 \leq 0.6 \mu A$ when JJ2 and JJL2 are on. The oscillation of switching current originates from the nonlocal Josephson effect derived from the coupling of JJ1 and JJ2. The curves shift along the $B$ axis as $I_2$ varies, implying that the phase shift of JJ2 ($\Delta \phi_2$) is induced. (B) Evaluated $\Delta \phi_2$ as a function of $I_2$ from (A). $\Delta \phi_2$ is tunable with $I_2$ in the range of $-0.2 \pi < \Delta \phi_2 < 0.2 \pi$. (C) The obtained JJ1 supercurrent as a function of $B$ at $-0.6 \mu A \leq I_2 \leq 0.6 \mu A$. Line colors specifying $I_2$ are consistent with those in (A). The curve shapes are modulated by changing $I_2$.  

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with \((V_{g1}, V_{g2}) = (-1.4 \text{ V}, -1.45 \text{ V})\), JJL1 off, and \(V_{g1,2} = -1.15 \text{ V}\). Under this condition, the switching current of JJ1 \((I_{sw1})\) depends on \(\phi_2\) due to the nonlocal Josephson effect derived from the AMSs \((28, 44)\).

\(I_{sw1}\) is displayed in Fig. 2A as a function of \(B\) for several \(I_2\) values between \(-600\) and \(600 \text{ nA}\). We note that \(I_2\) in the range is smaller than the switching current of the smaller SQUID. \(I_{sw1}\) oscillates with \(B\), originating from the coherent coupling between JJ1 and JJ2. The oscillation pattern gradually shifts along the \(B\) axis as \(I_2\) varied. This indicates that \(\Delta \phi_2\) is induced by \(I_2\), and the switching current is described as \(I_{sw1}(2\pi \Phi_0/B \Phi_0 + \Delta \phi_2(I_2))\). As the single oscillation period corresponds to \(2\pi\), \(\Delta \phi_2(2\pi/2\pi)\) is estimated from the \(I_{sw1}(2\pi \Phi_0/B \Phi_0 + \Delta \phi_2(I_2))\) curves as a ratio of the shift along the \(B\) axis from the curve of \(I_2 = 0 \text{ nA}\) to the single oscillation period in \(B\). The estimated \(\Delta \phi_2\) versus \(I_2\) is portrayed in Fig. 2B. In the range of \(-600 \text{ nA} \leq I_2 \leq 600 \text{ nA}\), \(\Delta \phi_2\) is tuned by \(-0.4\pi\).

Subsequently, we measure the CPR of JJ1 with \(JJ\) on by varying \(I_2\). The CPR curves are presented as a function of \(B\) in Fig. 2C. The result for \(I_2 = 0 \text{ nA}\) corresponds to the green data in Fig. 1G. The curve shape gradually varies with \(I_2\), reflecting \(I_{sc1}(\phi_1, \phi_2)\) in \(-\pi \leq \phi_1 \leq \pi\). The CPR is nearly a point-symmetric function as expected from the time-reversal relation of \(\phi_2\) with \(\phi_1\). We note that the amplitude of the JJ1 CPR at \(\phi_2 \approx \pi\) \((-0.1 \mu\text{A})\) is smaller than that at \(\phi_2 \approx 0\) \((-0.2 \mu\text{A})\) as seen in Fig. 3 (A to D). The amplitude at \(\phi_2 \approx \pi\) is also smaller than that with \(JJ\) off \((-0.2 \mu\text{A})\) of the black circles in Fig. 1G. Therefore, when \(B\) makes \(\phi_2 \approx \pi\) \((\text{mod } 2\pi)\), difference between the JJ1 supercurrent with \(JJ\) on and with \(JJ\) off becomes large. This is the reason why the discrepancy between \(I_{sc1}\) with JJ2 on and with JJ2 off becomes large. These \(B\) points correspond to \(\phi_2 \approx \pi\) \((\text{mod } 2\pi)\) (see Fig. S3C).

Then, the phase shift of \(\phi_2\) induced by \(I_2\) makes the remarkable modulation of the curves around the \(B\) points in Fig. 2C.

**DISCUSSION**

The observation of AJE means that the CPR of JJ1 is asymmetric to \(\phi_1 = 0\) at \(\phi_2 \neq 0, \pi\). The origin of the AJE or the asymmetric CPR has been associated with the AMS physics in the literature \((28)\). To discuss the origin, we consider the ABSs in the single JJ1 and JJ2 with \(\phi_2\) fixed in \([0, \pi]\). With no coherent coupling, the ABSs in the single JJ2 are constant with \(\phi_1\). When the ABS spectrum in JJ1 is symmetric with respect to \(\phi_1 = 0\), the energies monotonically decrease as \(\phi_1\) is swept away from \(\phi_1 = 0\) \((25)\). Then, the single ABSs in JJ1 can cross the single ABSs in JJ2 at two \(\phi_1\) points in \([0, \pi]\) and \([-\pi, 0]\). With the coherent coupling, the two ABSs are hybridized at the crossing points to open the energy gaps. At the crossing point in \([0, \pi]\), elastic cotunneling hybridizes the ABSs while crossed Andreev reflection does at the point in \([-\pi, 0]\). This difference induces the different energy gaps to cause the asymmetric Andreev spectrum to \(\phi_1 = 0\) in the coupled JJ1 (see note S7 and fig. S7). \(I_{sc1}\) is derived from the differential of the ABS energies with \(\phi_1\). Consequently, the CPR of JJ1 becomes asymmetric to \(\phi_1 = 0\), and the AJE can emerge.

The experimental and numerical results indicate that the oscillation amplitude of the JJ1 CPR at \(\phi_2 \approx \pi\) is smaller than that at \(\phi_2 \approx 0\) or that with no coherent coupling. Our additional calculation of the coupled JJ1s with the single conduction channel in each of JJ1s implies that the coupling energy of the JJ1 and JJ2 ABSs at \(\phi_2 \approx \pi\) is larger than that at \(\phi_2 \approx 0\). The larger coupling energy produces the larger modulation of the ABSs, resulting in the smaller \(I_{sc1}\) (see note S7 and fig. S7).

It may be valuable to refer to the SC diode effect in the coupled JJ1s \((45, 47, 48)\). The SC diode effect is evaluated from comparison of the positive \(I_{sw1}\) with the negative \(I_{sw1}\) at fixed \(\phi_2\). This corresponds to compare the maximum \(I_{sc1}\) with the minimum \(I_{sc1}\) in the obtained CPR of JJ1 at the \(\phi_2\). Ideally, the SC diode effect could be obtained from the two-dimensional CPR in Fig. 3A by the comparison. However, because of the noise in the data originating from the inductance and the \(\phi_1\) shift by \(I_2\) (see note S4), our CPR data do not have sufficient quality to reproduce the previously reported SC diode effect from the maximum and minimum \(I_{sc1}\) whose efficiency is typically small \((-5\%)\) \((45)\).
Las t, we explore $V_{g1}$ and $V_{g2}$ dependence of the two-dimensional CPR of JJ1. The evaluated JJ1 CPRs with several ($V_{g1}$, $V_{g2}$) are summarized in Fig. 4 (see note S4 and fig. S4). For example, the panel at the top-left corner shows $I_{sc1}(\phi_1, \phi_2)$ at ($V_{g1}$, $V_{g2}$) = (−1.4 V, −1.95 V). The JJ1 CPR results at the same $V_{g1}$ but different $V_{g2}$ are exhibited in the same rows with the same color scales. Figure 4 indicates that the AJE behavior decreases as $V_{g2}$ is made more negative, while the behavior is less modulated by $V_{g1}$. This gate dependence can be assigned to the relation of the numbers of ABSs in JJ1 and JJ2. In the case that the number of ABSs in JJ1 is smaller than that in JJ2, all the ABSs in JJ1 are coupled with the ABSs in JJ2. Therefore, the AJE behavior can remain evident. On the other hand, in the opposite case, some ABSs in JJ1 are not coupled with the ABSs in JJ2. Consequently, the AJE behavior becomes smaller as the number of ABSs in JJ2 decreases. This gate voltage dependence can be reproduced in our numerical calculation (see note S6 and fig. S6). Then, the gate dependence supports that the AJE in JJ1 emerges from the AMSs formed in the coupled JJs. We note that the AJE appears even in $V_{g1} = −1.7$ V where $I_{sc1}$ is of the order of 10 nA. This means that the AJE behavior in JJ1 is not an artifact from our evaluation method in which we assume that $\Phi_1$ and $\Phi_2$ are linearly dependent on $B$ and ignore the inductance term related to the circulating supercurrent in the SC loop because the small supercurrent makes the inductance effect smaller (see note S4 and fig. S4).

In our consideration, we do not include the spin-orbit interactions that the InAs quantum well holds. At least, the obtained AJE

### Fig. 3. The CPR of JJ1 coupled to JJ2 and the phase-tunable AJE.

(A) The obtained two-dimensional CPR of JJ1 coupled to JJ2 is shown. The JJ1 supercurrent depends not only on $\phi_1$ but also on $\phi_2$, which means that the coherent coupling of JJ1 and JJ2 produces the nonlocal Josephson effect. A purple line on $I_{sc1}(\phi_1, \phi_2) = 0$ nA is extended from $(\phi_1, \phi_2) = (0,0)$, exhibiting the evolution of $\phi_1$ for the ground state of JJ1 with $\phi_2$. Therefore, $\phi_1 \neq 0$ on the purple line indicates that the $\phi$ junction is formed and tunable by $\phi_2$. (B) Numerically calculated CPR of JJ1 coupled to JJ2 using the tight-binding model. This gives a good agreement with the experimental result in (A). (C) Line profiles at $\phi_1 = 0, \pm \pi/2, \pi$ in (A) are shown. (D) Line profiles at $\phi_1 = 0, \pm \pi/2, \pi$ in (B) are shown. (E) A line profile at $\phi_1 = 0$ in (A) is shown. The finite supercurrent flowing in JJ1 with $\phi_1 = 0$ and the spontaneous supercurrent is tunable with $\phi_2$. 
can be explained only by the coherent coupling, and it is difficult to
discuss roles of the spin-orbit interactions in these results. To elu-
cidate such physics, it may be useful to study the SC transport in the
coupled JJs with the in-plane magnetic fields because the in-plane
magnetic fields lifting the spin degeneracy produce the various SC
phenomena related to the spin-orbit interactions in the single JJs
such as the SC phase batteries, SC diodes, and Majorana zero modes.

In conclusion, we construct a two-dimensional CPR of a JJ co-
herently coupled to another JJ using asymmetric SQUIDs. From the
CPR, we demonstrate the spontaneous supercurrent and $\phi$ junction
controlled by the nonlocal phase difference, indicating the phase-
tunable AJE. The obtained AJE contributes to the development of
functional SC devices such as SC phase batteries and SC diodes. Our
method for the two-dimensional CPR evaluation will be applicable
to multiterminal JJs as well, where a single normal metal is coupled
to several SC electrodes (21, 59–65).

**MATERIALS AND METHODS**

**Sample growth**

The wafer structure has been grown via molecular beam epitaxy
on a semi-insulating InP substrate. The stack materials from bottom
to top are a 100-nm In$_{0.52}$Al$_{0.48}$As buffer, a five-period 2.5-nm
In$_{0.53}$Ga$_{0.47}$As/2.5 nm In$_{0.52}$Al$_{0.48}$As superlattice, a 1-μm-thick
metamorphic graded buffer stepped from In$_{0.52}$Al$_{0.48}$As to
In$_{0.84}$Al$_{0.16}$As, a 33-nm graded In$_{0.84}$Al$_{0.16}$As to In$_{0.81}$Al$_{0.19}$As
layer, a 25-nm In$_{0.81}$Al$_{0.19}$As layer, a 5-nm InAs quantum well, a 10-nm In$_{0.81}$Ga$_{0.19}$As top barrier, two monolayers of GaAs, and
lastly, an 8.7-nm layer of epitaxial Al. The top Al layer has been grown in the same
chamber without breaking the vacuum. The two-dimensional
electron gas (2DEG) accumulates in the InAs quantum well.

**Device fabrication**

In this study, conventional electron beam lithography was used to
fabricate JJs. We etched out the aluminum film using the type D
etchant after we defined the mesa of the InAs quantum well with
1:1:8 of H$_3$PO$_4$:H$_2$O$_2$:H$_2$O etchant. Subsequently, we grew a 30-
nm-thick Al$_2$O$_3$ film by atomic layer deposition and deposited Ti
and Au to make the gate electrodes.

**Measurement**

For the measurement of the switching current, we measured the $I$-$V$
curves of JJ1 for various conditions. When switching JJL1 or JJL2 off
to measure the single or coupled JJ1, we set $V_{g1} \leq -1.9$ V or $V_{g2} \leq
-2.0$ V. When switching JJL1 or JJL2 on to form the asymmetric
SQUIDs, we set $V_{g1} = -1.4$ V or $V_{g2} = -1.15$ V. The switching
currents of JJL1 and JJL2 at $V_{g1} = -1.4$ V and $V_{g2} = -1.15$ V
are 0.8 and 1.0 μA, respectively. When pinching off JJL1 and JJL2,
we set $V_{g1}$ ($V_{g2}$) $= -4$ V.

![Fig. 4. Gate voltage dependence of the two-dimensional CPR of JJ1.](https://www.science.org/)

The JJ1 CPR $I_{sc1}(\phi_1, \phi_2)$ results obtained at several sets of ($V_{g1}, V_{g2}$) are shown. The panels in the
same row (column) are obtained at the same $V_{g1}$ ($V_{g2}$), labeled on the right side (upside). In addition, the same row images are depicted with the same color scales placed
on the right side. These results indicate that the dependence on $\phi_2$ becomes weaker as $V_{g2}$ becomes more negative while it is almost unchanged though $V_{g1}$ is varied.
REFERENCES AND NOTES


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